MASTER COPY KEEP THIS COPY FOR REPRODUCTION PURPOSES

AD-A226 064

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

2

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	1990	Final 13 Jun 88 - 12 Jun 89

4. TITLE AND SUBTITLE

Morphology Workshop

5. FUNDING NUMBERS

DAAL03-88-G-0035

AUTHOR(S)

Robert M. Haralick (Principal Investigator)

PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

University of Washington
Seattle, WA 98195

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

ARO 26131.1-EL-CF

11. SUPPLEMENTARY NOTES

The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Mathematical morphology provides the operations of dilation, erosion, opening and closing for performing non-linear image analysis for binary or grayscale images. The algebra of mathematical morphology is as rich as the algebra of convolution and correlation. The algebra is not based on linear combination and has no relation to spatial frequencies. Rather, it has an intrinsic connection to shape due to the primitive matching property inherent in the opening and closing morphological operations.

Mathematical morphology can be an important component of many of the future smart sensors the Army is developing. The technology of mathematical morphology has proved

(Abstract continued on reverse side)

14. SUBJECT TERMS

Mathematical Morphology, Morphology, Nonlinear Image Analysis Image Analysis, Smart SEnsors, Workshop

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

90 08 28 027

itself in the manufacturing industry where it is being successfully used on the factory floor for visual robot guidance, object recognition, inspection, and flaw detection. Almost every company manufacturing pixel pushing machine vision boards has one board capable of performing mathematical morphology as well as the traditional convolution operations.

It is now time for the Army to benefit from what morphology work has already been done and to develop a program of what needs to be done. The purpose of the morphology workshop was to review some of the basic and existing theory of morphology, illustrate how morphology performs recognition shape extraction, present new results in mathematical morphology, and make recommendations to the Army about new research areas in morphology which need to be developed in order to benefit Army programs. The workshop involved invited speakers who ar known for the work they have published or done in mathematical morphology.

ARO 26131.1-EL-CF

Morphology Workshop

Final Report

Robert M. Haralick

Department of Electrical Engineering, FT-10
Universty of Washington
Seattle, WA 98195

Project P.26131-EL-CF

Grant DAAL03-88-G-0035
U.S. Army Research Office

A

1. Summary

Mathematical morphology provides the operations of dilation, erosion, opening and closing for performing non-linear image analysis for binary or grayscale images. The algebra of mathematical morphology is as rich as the algebra of convolution and correlation. The algebra is not based on linear combination and has no relation to spatial frequencies. Rather, it has an intrinsic connection to shape due to the primitive matching property inherent in the opening and closing morphological operations.

Mathematical morphology can be an important component of many of the future smart sensors the Army is developing. The technology of mathematical morphology has proved itself in the manufacturing industry where it is being successfully used on the factory floor for visual robot guidance, object recognition, inspection, and flaw detection. Almost every company manufacturing pixel pushing machine vision boards has one board capable of performing mathematical morphology as well as the traditional convolution operations.

It is now time for the Army to benefit from what morphology work has already been done and to develop a program of what needs to be done. The purpose of the morphology workshop was to review some of the basic and existing theory of morphology, illustrate how morphology performs recognition shape extraction, present new results in mathematical morphology, and make recommendations to the Army about new research areas in morphology which need to be developed in order to benefit Army programs. The workshop involved invited speakers who are known for the work they have published or done in mathematical morphology.

2. Workshop Overview

The two-day workshop was held under the auspices of MICOM at the Tom Bevill Center of the University of Alabama, Huntsville, on 25 and 26 July 1988. The invited presenters were

Dr. Robert M. Haralick
University of Washington

Dr. Stephen S. Wilson
Applied Intelligent Systems, Inc.

Dr. Petros Maragos
Harvard University

Dr. Ronald W. Schafer
Georgia Tech

Seventeen people attended the workshop.

3. Workshop Program

Program
25 July 1988

08:00–09:30	*Image Analysis Using Morphology: The Concepts,* Robert M. Haralick
09:30–10:30	*Applications of Morphology in Industry: Part I,* Steve Wilson
10:30–10:45	*Coffee Break*
10:45–11:45	*Applications of Morphology in Industry: Part II,* Steve Wilson
11:45–13:00	*Lunch*
13:00–14:30	*Image Analysis Using Morphology: The Algebra,* Robert M. Haralick
14:30–15:45	*Army Needs and Morphology: Panel Session I*
15:45–16:00	*Coffee Break*
16:00–17:00	*Morphological Signal Processing Systems: Part I,* Ron Schafer

Program
26 July 1988

08:00–09:00	*Mathematical Morphology Applied to Multi-Scale Image Representation and Shape Description,* Petros [illegible]ragos
09:00–09:30	*Morphological Signal Processing Systems: Part II,* Ron Schafer
09:30–10:30	*Army Needs and Morphology: Panel Session II*
10:30–10:45	*Coffee Break*
10:45–11:15	*Mathematical Morphology Applied to Multi-Scale Image Representation and Shape Description: Part II,* Petros Maragos
11:15–11:30	*Closing Remarks*

4. Abstracts of Talks

Image Analysis Using Mathematical Morphology

Robert M. Haralick
Intelligent Systems Laboratory
Department of Electrical Engineering • FT-10
University of Washington
Seattle, WA 98195

ABSTRACT

For the purposes of object or defect identification required in industrial vision applications, the operations of mathematical morphology are more useful than the convolution operations employed in signal processing because the morphological operators relate directly to shape. This talk reviews both binary morphology and grayscale morphology, covering the operations of dilation, erosion, opening and closing, their relations and the morphological sampling theorem. Examples are given for each morphological concept and explanations are given for many of their inter-relationships. A comparison between the algebra of morphology and the algebra of convolution reveals some important similarities which are suggestive of the underlying depth and richness of the non-linear algebra of morphology.

Applications of Morphology in Industry

Steve Wilson
Applied Intelligent Systems, Inc.
Ann Arbor, MI 48103

ABSTRACT

Morphology has been successfully used in a number of applications in industrial machine vision. This session will outline vision principles involved in the application of morphology to several different real world image processing problems such as classification of different objects, location and dimensional measurement of objects to subpixel accuracy, and texture analysis such as flaws in machined or painted surfaces. Familiarity with image processing will be helpful but not necessary.

This session will not involve a rigorous math approach, but instead its emphasis will be on gaining an intuition on how the application of various grayscale and binary technique can successfully handle images encountered in adverse envirionments where conditions cannot be well controlled.

Many of today's machine vision hardware systems can be programmed to take advantage of the methods discussed, which will range from simple applications involving erosion, dilation, and skeletonizing to newer morphological techniques involving vector correlation and majority voting logic. Other topics to be covered will include applications where binary morphology fails, where linear methods will fail, how to do thresholding and segmentation correctly, tradeoffs between processing time and robustness, how to handle moving images, variation in illumination levels and textured backgrounds, finding various image features, and the transition from morphology to higher level processing concepts.

For each of the important application categories, a number of slides will be shown illustrating step by step morphology processing from the input camera image to the final recognition.

Morphological Signal Processing Systems

C. H. Richardson and R. W. Schafer
Georgia Institute of Technology
School of Electrical Engineering
Atlanta, Georgia 30332

ABSTRACT

This talk will begin by reviewing some important relationships between morphological systems and a variety of signal processing transformations that were originally developed outside the framework of morphological system theory. This discussion is intended to highlight the need for a systematic approach to the design of signal processing systems that are based on the principles and fundamental representational theorems of mathematical morphology. A second aspect of the talk will be concerned with some applications of morphological signal processing systems. The talk will conclude with a general discussion of the some of the fundamental problems in computer-aided design and analysis of morphological systems. Specifically discussed will be preliminary research on developing a LISP-based signal processing environment for the representation of discrete signals and systems for both symbolic and numeric manipulations of morphological systems.

Mathematical Morphology Applied to Multi-Scale Image Representation and Shape Description

Petros Maragos
Division of Applied Sciences
Harvard University
Cambridge. MA 02138

ABSTRACT

Two fundamental problems in computer vision are how to represent image objects at multiple scales and how to describe their shapes. Mathematical morphology is a formal and quantitative methodology to image analysis, which directly extracts information about the shape and size of image objects. As such, it can offer a systematic approach to solving the above two problems. Specifically we will discuss the use of morphological skeletons for a very general and versatile multi-scale image representation transform; morphological openings for nonlinear multi-scale image smoothing and a derived shape-size descriptor, the pattern spectrum; a shape-size approach for a symbolic image modeling by parts; and the use of morphological skeletonization for modeling fractal images.

Through successive erosions and openings of an image with respect to an arbitrary structuring element, a sequence of small critical image parts can be obtained, called skeleton components, whose superposition is the morphological skeleton. The ensemble of all skeleton components can exactly reconstruct the image through dilations. Elimination of some components is equivalent to morphological opening (smoothing) the image at a scale equal to the number of eliminated components. By also varying the structuring element, multi-shape multi-scale structural distributions in images can be modeled by skeletons and openings with direct applications to data compression and progressive image transmission.

The openings can also complement linear smoothing filters used in multi-scale image analysis, because openings can suppress noise without shifting or blurring image edges and axiomatize the concept of scale. In addition, areas of differences among successive openings create a useful shape-size descriptor, the pattern spectrum, which can detect critical scales in images.

Morphological concepts can be used to rigorously formulate a symbolic image modeling problem. Here the image is modeled as a nonlinear superposition of simpler parts (the "symbols"), which are translated and scaled shape patterns (structuring elements) drawn from a finite collection. Then

the model parameters can be found by using the information from openings and pattern spectrum, and via local searches at points of generalized skeletons. This symbolic modeling appears promising in bridging the gap between low-level image processing operations and high-level vision tasks such as object recognition.

Finally, in the theory of iterated function systems, a fractal image can be modeled arbitrarily closely as the attractor of a finite set of affine maps. We use the morphological skeleton to efficiently extract the parameters of these affine maps. This technique has applications for fractal image analysis/synthesis, computer graphics, and coding.

5. Discussion at Workshop

Remarks on Morphology Workshop Panel Discussion
by Stephen Dow

The morphology workshop panel discussion brought out a number of issues relating morphology to Army image processing applications. Some of the general types of processing where morphology was identified as being applicable are image enhancement or filtering, shape feature extraction, and data compression. Some concern was expressed that the shape detection capabilities of the morphological operations may be more applicable to images from manufacturing applications than to those from Army applications because in the former case the shapes of interest tend to be more predictable and well-defined than in the latter. It may be the case that the complex imagery involved in automatic target recognition applications precludes the free-form, interactive selection of morphological operations and structuring elements which will extract desired objects; techniques for automated selection and optimization may be necessary. It was pointed out that mathematical morphology is not a stand-alone method but a set of tools to be used along with other methods. Even if morphological operations cannot directly extract the objects of interest there are probably portions of the processing which can be aided by these operations and by the algebraic methodology morphology provides.

There was also some discussion relating to the existence and availability of image data bases and criteria for performance evaluation. These factors are important to the research community's ability to develop, test, and demonstate the applicability of morphological methods to Army imagery.

APPLICATIONS OF MORPHOLOGY IN INDUSTRY

Stephen S. Wilson, Applied Intelligent Systems, Inc.

Many of the tools of morphology have been successfully applied in industrial applications. The erosion, dilation opening and closing operations are useful for noise filtering and for recognizing and locating simple shapes defined by a structuring element. Also, a wide variety of more complex but useful tools fall under the heading of "hit or miss" operations such as topological filters, convex hull, skeletonizing, feature finding, conditional dilation, and directional filtering. Generalization of the above operations to fuzzy logic is one way of extending the morphology concepts so that they can be directly applied to grey level images. Other generalizations such as majority voting logic are useful when applied to noisy images.

Vector correlation is similar to the usual concept of correlation, but where the picture and kernel are vectors multiplied using the inner product. In morphology, a vector structuring element can be defined where the basis of the vector space is a number of feature bit planes. Thus, morphology can be used for relating features to classify objects.

Industrial applications largely fall into five application categories:
1. Classification, such as character recognition using vector morphology,
2. Location; finding overlapping parts to subpixel accuracy using vector correlation,
3. Texture defects, such as finding flaws in TV screens by directly applying openings and closings,
4. Dimensional measurement, where morphology is used to locate positions where measurement is to take place, and
5. Flaw detection, such as missing, or improperly punched holes in metal parts.

There are two broad classes of parallel computer architecture used in morphological image processing: MIMD (Multiple Instruction, Multiple Data) systems which are coarse grained, and SIMD (Single Instruction, Multiple Data) systems which have fine grained processing elements. The most popular types of MIMD systems are pipeline processors where a large number of functional modules can be interconnected to form a real time image processing system. Examples of these systems are the ERIM Cytocomputer, and a large variety of cards manufactured by Datacube which can be configured using their proprietary Maxvideo buss. However, these systems can be bulky and expensive, and difficult to configure in the field.

The most popular type of SIMD architecture is the mesh connected system where a large number of simple processing elements are connected in an NxN two dimensional grid such as in the GAPP chip manufactured by NCR. Although these systems are superb at morphology, the data I/O tends to be quite complex, and existing systems are large and expensive. The Applied Intelligent Systems Inc. AIS-5000 is an industrial system with a one dimensional array of moderately complex processors. The Centipede is an evolution of the AIS-5000 which is small, low cost, and more powerful, and is a candidate for an automatic target recognition system.

6. Letters

al Aeronautics and
Space Administration

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama
35812

NASA

Reply to Attn of: Mail Stop EB44
MSFC, AL 35812

August 1, 1988

Dr. Robert M. Haralick
Boeing Clairmont Egtvedt Professor
Department of Electrical Engineering
University of Washington
402 Electrical Engineering Building FT-10
Seattle, Washington 98195

Dear Dr. Haralick,

The workshop last week has aroused interest in the potential that morphology shows for use on several NASA programs. Projects at the Marshall Center that may benefit from its application include: vision sensor driven off-line programming of a robot for Solid Rocket Booster refurbishment; vision guided rendezvous and docking for an autonomous satellite service vehicle; and the capture of a CAD data base via the use of 3D Computed Tomography CT data. The last application, in particular, is interesting since it would require the extension of morphological techniques to 3D images, and it could have widespread use in industry.

Thanks again for organizing such an interesting and timely conference. I look forward to receiving the post-workshop report.

Sincerely

Ken Fernandez

Ken Fernandez, Ph.D.
Information and Electronic
Systems Laboratory

TECHNICAL LETTER KN11-ADVTEC-HV-1-0080

TELEDYNE BROWN ENGINEERING

Contract DASG60-87-C-0042
TA 150
CDRL A001

TO: Advanced Technology Directorate
U.S. Army Strategic Defense Command
P.O. Box 1500
Huntsville, Alabama 35807

Attention: Mr. Doyce Satterfield, CSSD-H-V

FROM: Teledyne Brown Engineering
SETAC Project Office
Cummings Research Park
Huntsville, Alabama 35807

Author: Wendell A. Childs

UBJECT: Morphology Workshop

ATE: 1 August 1988

1) On 25 and 26 July I attended the Morphology Workshop at the Beville Center, University of Alabama in Huntsville. The handout material from the Workshop is attached. Additional material to include a copy of viewgraphs used in the workshop and list of attendees will be supplied at a later date.

2) Morphology in a nonlinear mathematical formalism that is aptable to parallel digital processing. It has potential for plications in image analysis, target recognition and discrimination. Work is needed to determine just how well it can satisfy the requirements of these applications. Examples of the use of morphology to good advantage in industry were described during the Workshop and were very impressive. If the opportunity should arise it appears using morphology algorithms in the discrimination process would be a fertile area for investigation.

Wendell A. Childs

Wendell A. Childs
Advanced Technology

7. Recommendations

It is clear from the discussions at the workshop that mathematical morphology is an important image analysis tool that has applications not only in industry but also in smart sensor systems. This is confirmed by the letters of section 6. It would be worthwhile for some basic and DOD applied research to be sponsored in this area.

8. List of Attendees

J.S. Bennett
MICOM/AMSMI-RD-RE
U.S. Army Missile Command
Redstone Arsenal, AL 35898-5253
(205) 876-1623

J.L. Johnson
MICOM/AMSMI-RD-RE-op
U.S. Army Missile Command
Redstone Arsenal, AL 35898-5253
(205) 876-3820

Scott Speigle
MICOM/AMSMI-RD-GC-C
U.S. Army Missile Command
Redstone Arsenal, AL 35898-5253
(205) 876-8281

Judy Denton
NASA/EB44
MSFC. AL 35812
(205) 544-3831

Stephen Dow
Department of Mathematics
University of Alabama
Huntsville, AL 35899
(205) 895-6470

C.C. Sung
Department of Physics
University of Alabama
Huntsville, AL 35899
(205) 895-6117

Wendell Childs
Teledyne Brown Engineering
300 Sparkman Drive
Huntsville, AL 35807
(205) 532-8435

William Pittman
MICOM/AMSMI-RD-AS-PM
U.S. Army Missile Command
Redstone Arsenal, AL 35898-5253
(205) 876-1778
(AV) 746-1778

Henry C. Hollman
CSSD-H-SAI
U.S. Army SDC
P.O. Box 1500
Hunstville, AL 35807
(205) 895-5459

Stephen Wilson
Applied Intelligent Systems, Inc.
110 Parkland Plaza
Ann Arbor, MI 48103
(313) 663-8051

Petros Maragos
Harvard University
Division of Applied Sciences
Cambridge, MA 02138
(617) 495-4390

Craig Richardson
Georgia Tech
P.O. Box 31431
Atlanta, GA 30332
(404) 894-2910 X-1

Ronald W. Schafer
Georgia Tech
School of Electrical Engineering
Atlanta, GA 30332
(404) 894-2917

Ken Fernandex
NASA-EB44
MSFC, AL 35812
(205) 544-3825

Jim Epperson
Department of Mathematics
University of Alabama
Hunstville, AL 35899
(205) 895-6611

Richard Sims
MICOM/AMSMI-RD-AS-SS
U.S. Army Missile Command
Redstone Arsenal, AL 35898-5253
(205) 876-1648/1879

Larry Z. Kennedy
Applied Research, Inc.
P.O. Box 11220
Huntsville, AL 35814
(205) 837-8600

Part of ARO 26131.1-EL-CF

Morphological Signal Processing Systems

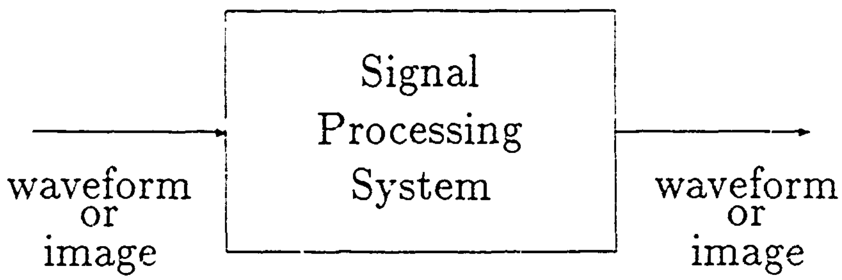
Craig H. Richardson and Ronald W. Schafer

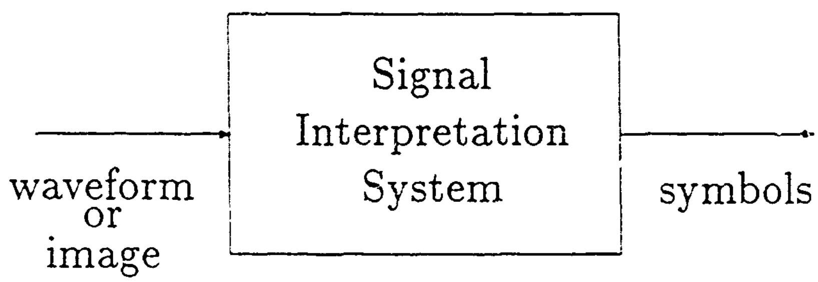
Georgia Institute of Technology

School of Electrical Engineering

Atlanta, Georgia 30332

Signal Processing





- Design of signal processing systems may be facilitated by symbolic manipulation of signal processing expressions.

Basic Set Operations

Shifted Set: $(X)_b = \{z : z = x + b; x \in X\}$

Complement Set: $(X)^c = \{z : z \notin X\}$

Symmetric Set: $\check{B} = \{z : z = -b; b \in B\}$

Minkowski Sum:

$$\begin{aligned} X \oplus B &= \{z : z = x + b; x \in X, b \in B\} \\ &= \bigcup_{b \in B} (X)_b = \bigcup_{x \in X} (B)_x = X \oplus B \\ &= \{z : X \cap (\check{B})_z \neq \emptyset\} \end{aligned}$$

Minkowski Difference:

$$\begin{aligned} X \ominus B &= (X^c \oplus B)^c = \bigcap_{b \in B} (X)_b \\ &= \{z : (\check{B})_z \subseteq X\} = \{z : z - b \in X; b \in B\} \end{aligned}$$

Duality: $X \ominus B = (X^c \oplus B)^c$; $X \oplus B = (X^c \ominus B)^c$

Basic Morphological Systems

Dilation: $\mathcal{D}(X, B) = X \oplus \check{B} = \{z : X \cap (B)_z \neq \emptyset\}$

Erosion: $\mathcal{E}(X, B) = X \ominus \check{B} = \{z : (B)_z \subseteq X\}$

Duality:

$$\begin{aligned}
\mathcal{E}(X, B) &= X \ominus \check{B} = (X^c \oplus \check{B})^c = [\mathcal{D}(X^c, B)]^c \\
\mathcal{D}(X, B) &= X \oplus \check{B} = (X^c \ominus \check{B})^c = [\mathcal{D}(X^c, B)]^c
\end{aligned}$$

Opening: $\mathcal{O}(X, B) = (X \ominus \check{B}) \oplus B = \mathcal{D}(\mathcal{E}(X, B), \check{B})$

Closing: $\mathcal{C}(X, B) = (X \oplus \check{B}) \ominus B = \mathcal{E}(\mathcal{D}(X, B), \check{B})$

Duality: $\mathcal{O}(X, B) = [\mathcal{C}(X^c, B)]^c$; $\mathcal{C}(X, B) = [\mathcal{O}(X^c, B)]^c$

Open-Closing: $\mathcal{OC}(X, B) = \mathcal{C}(\mathcal{O}(X, B), B)$

Close-Opening: $\mathcal{CO}(X, B) = \mathcal{O}(\mathcal{C}(X, B), B)$

(**Note:** For symmetric structuring elements, $\check{B} = B$, so we can drop the ˇ and all definitions of morphological operators give the same results.)

Properties of Morphological Systems

Increasing: For example,

$$\text{If } X_1 \subseteq X_2, \text{ then } \mathcal{E}(X_1, B) \subseteq \mathcal{E}(X_2, B)$$

Translation Invariant: For Example,

$$\mathcal{D}((X)_z, B) = [\mathcal{D}(X, B)]_z$$

Extensive/Antiextensive:

$$\mathcal{E}(X, B) \subseteq \mathcal{O}(X, B) \subseteq X \subseteq \mathcal{C}(X, B) \subseteq \mathcal{D}(X, B)$$

Idempotence:

$$\mathcal{O}(\mathcal{O}(X, B), B) = \mathcal{O}(X, B)$$

$$\mathcal{C}(\mathcal{C}(X, B), B) = \mathcal{C}(X, B)$$

Systems for Thinning - I

Hit or Miss Operator:

Assume $B = (B_1, B_2)$ where B_1 and B_2 are disjoint sets.

$$
\begin{aligned}
\mathcal{H}(X, B) &= (X \ominus \check{B}_1)/(X \oplus \check{B}_2) \\
&= \{z : z \in (X \ominus \check{B}_1) \text{ and } z \notin (X \ominus \check{B}_2)\} \\
&= (X \ominus \check{B}_1) \cap (X \oplus \check{B}_2)^c = (X \ominus \check{B}_1) \cap (X^c \ominus \check{B}_2) \\
&= \mathcal{E}(X, B_1) \cap \mathcal{E}(X^c, B_2)
\end{aligned}
$$

The hit or miss operator selects the set of points for which B_1 is inside the image and B_2 is inside the background.

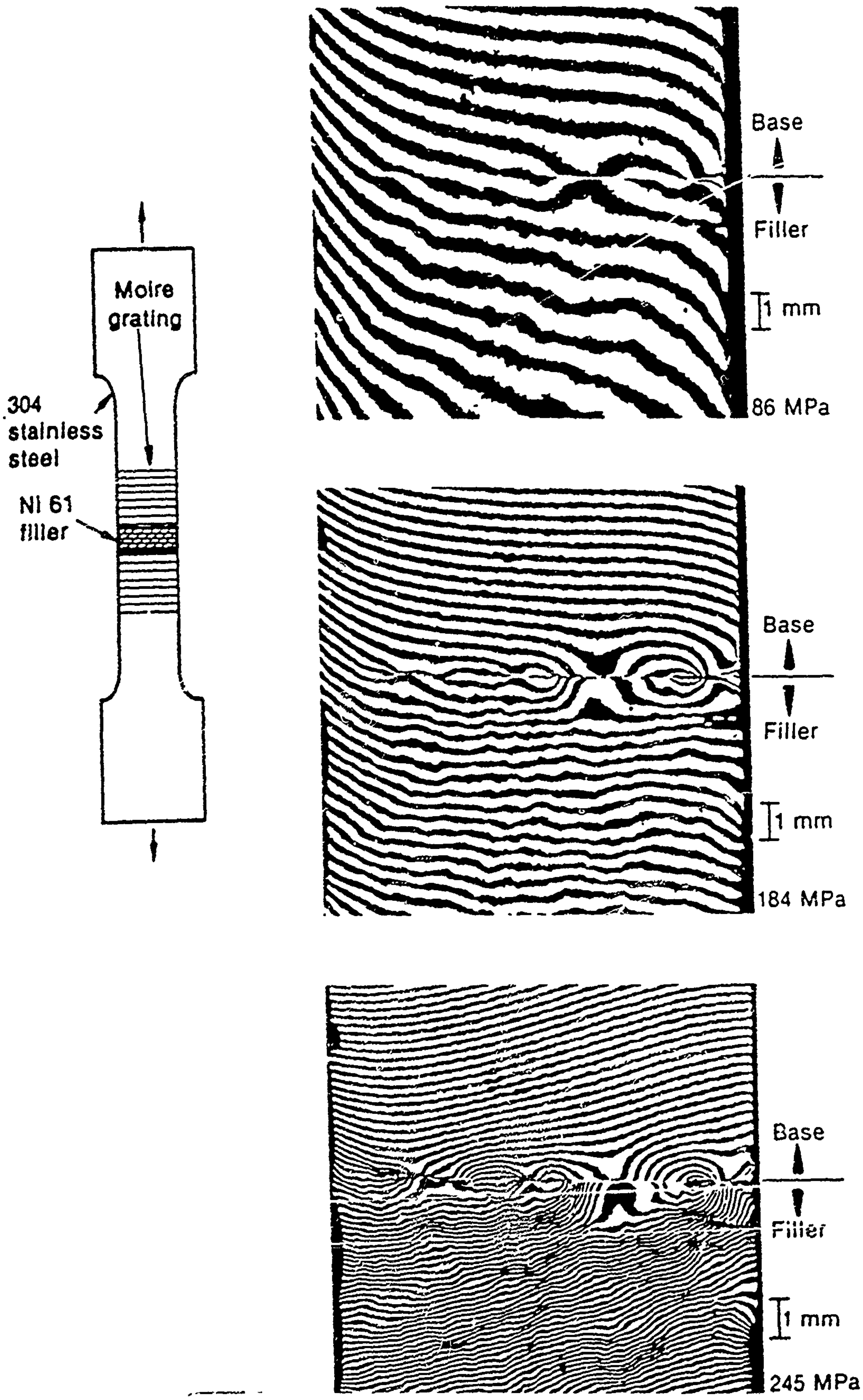
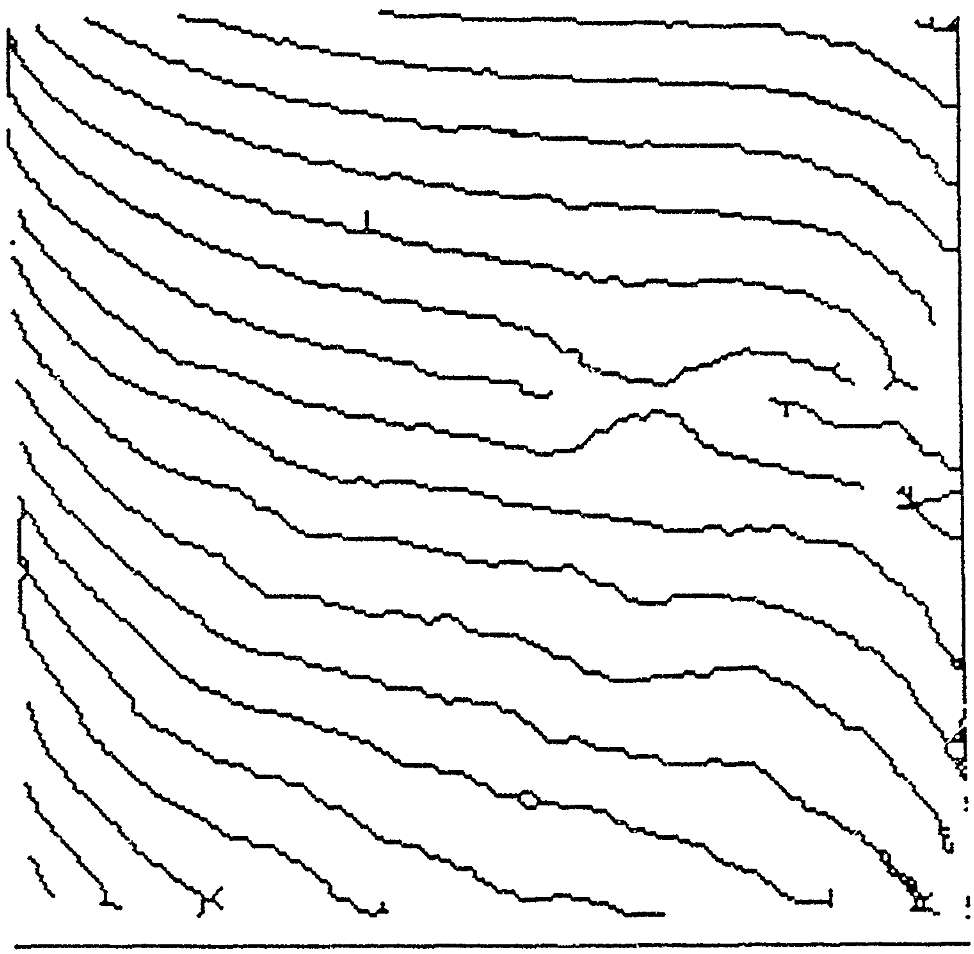


Figure 7b. Moire interferometry displacement field of a lack of fuison weld defect. The net in-plane displacement per fringe is 0.4 microns.

8 0888



Systems for Thinning - II

Skeleton:

The skeleton $S(X)$ of a binary image is defined as

$$S_n(X) = \mathcal{E}(X, nB)/\mathcal{O}(\mathcal{E}(X, nB), B)$$

for $n = 0, 1, \ldots, N$.

$$S(X) = \bigcup_{n=0}^{N} S_n(X)$$

The original image can be reconstructed from the skeleton subsets by

$$X = \bigcup_{n=0}^{N} \mathcal{D}(S_n(X), n\check{B})$$

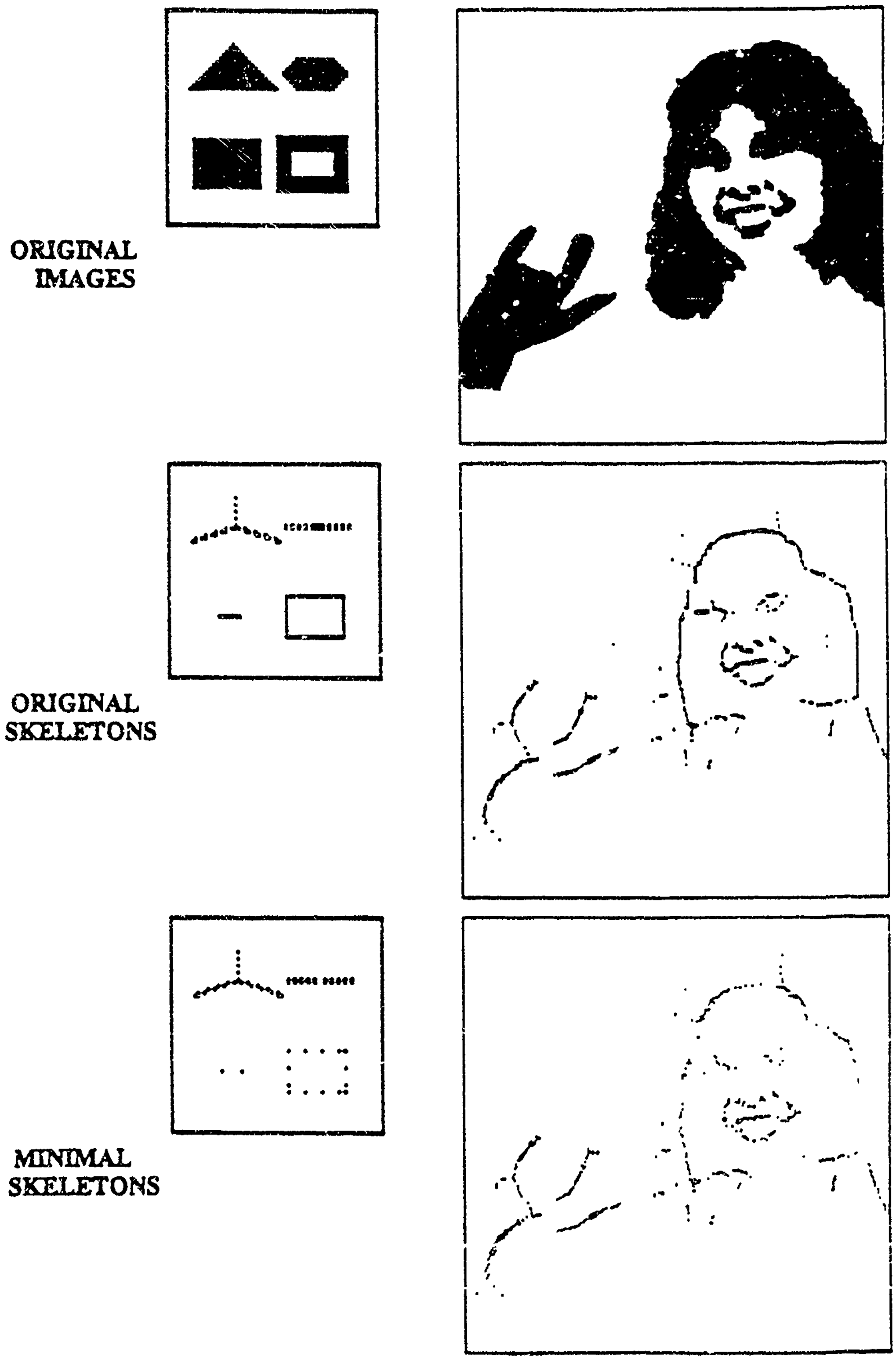
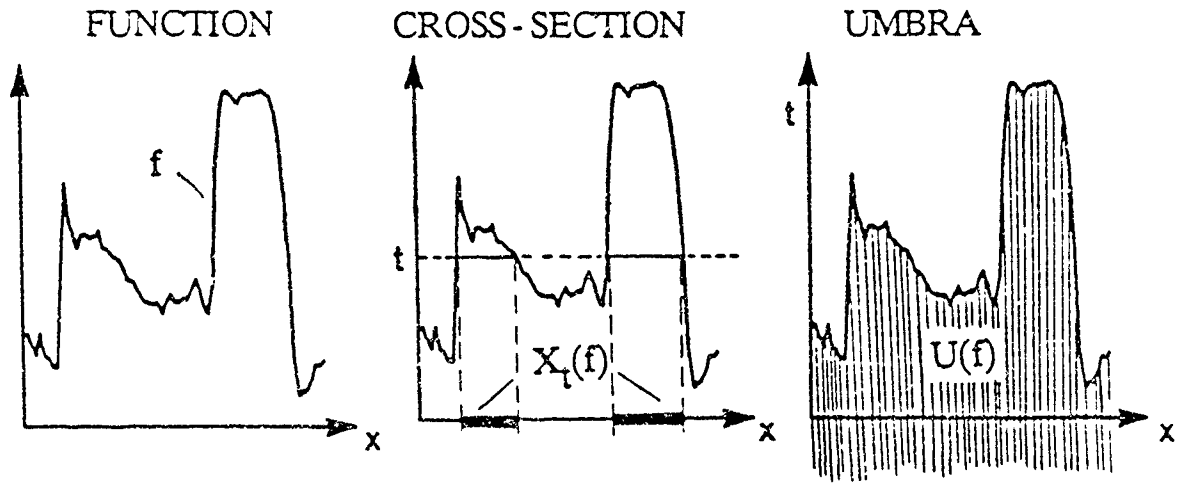


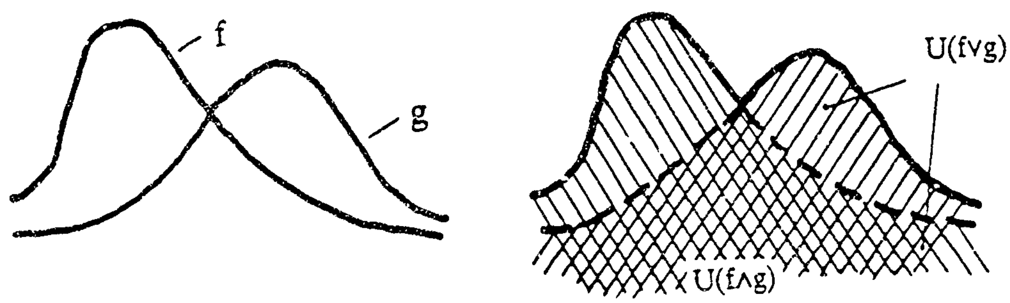
Figure 6-6. Images, skeletons, and globally minimal skeletons (struct. element = SQUARE).

Representing Functions by Sets

- Functions can be represented by sets.



- min and max are isomorphic to intersection and union.



$$(f \wedge g)(x) = \min[f(x), g(x)] \iff U(f) \overset{\cap}{\cancel{\cup}} U(g)$$

$$(f \vee g)(x) = \max[f(x), g(x)] \iff U(f) \overset{\cup}{\cancel{\cap}} U(g)$$

$$f(x) \leq g(x) \quad \forall x \iff U(f) \subseteq U(g)$$

Morphological Systems for Functions

Minkowski Sum:

$$(f \oplus g)(x) = \max_z [f(z) + g(x - z)]$$

Minkowski Difference:

$$(f \ominus g)(x) = \min_z [f(z) - g(x - z)]$$

Dilation:

$$\mathcal{D}(f, g)(x) = (f \oplus \hat{g})(x) = \max_z [f(z) + g(z - x)]$$

$$\hat{g}(x) = g(-x) \qquad \text{(reflected function)}$$

Erosion:

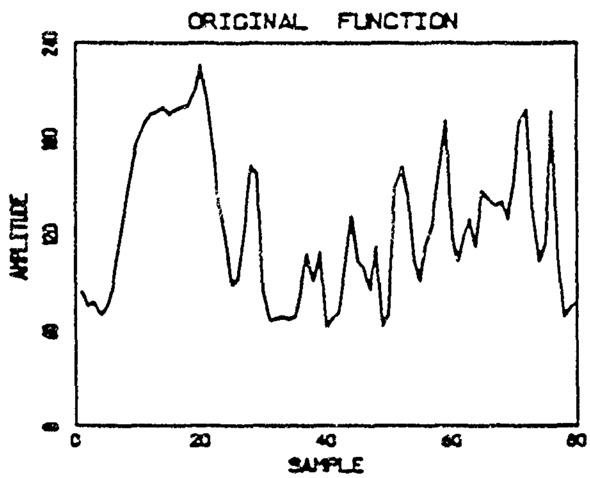
$$\mathcal{E}(f, g)(x) = (f \ominus \hat{g})(x) = \min_z [f(z) - g(z - x)]$$

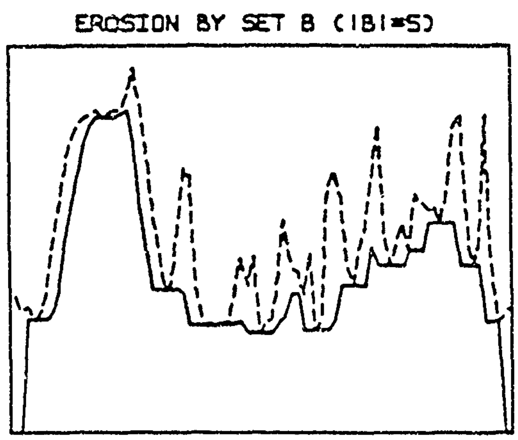
Opening and Closing:

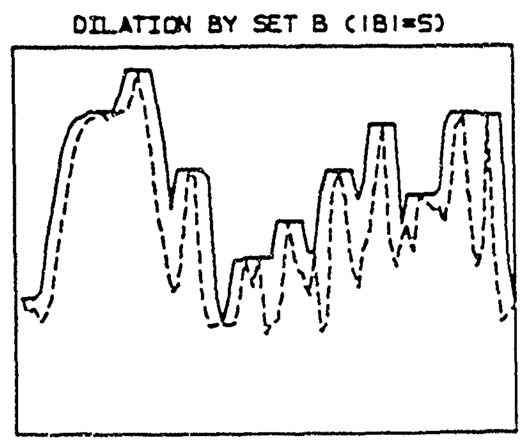
$$\mathcal{O}(f, g) = \mathcal{D}(\mathcal{E}(f, g), \hat{g}) \qquad \mathcal{C}(f, g) = \mathcal{E}(\mathcal{D}(f, g), \hat{g})$$

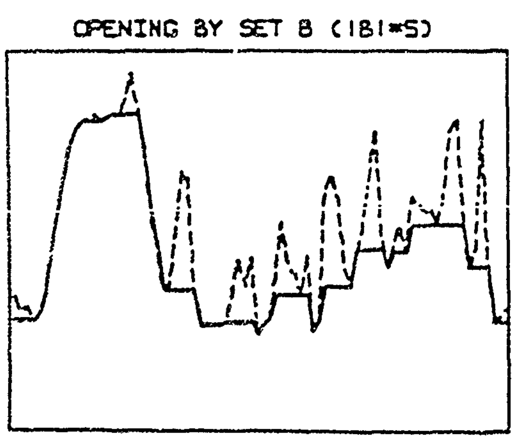
Morphological Filtering

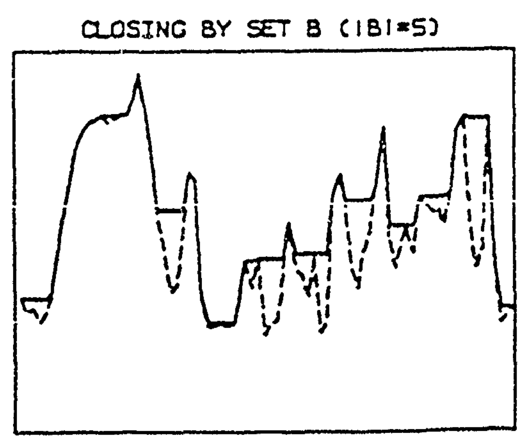
- Morphological systems can be used to smooth functions.







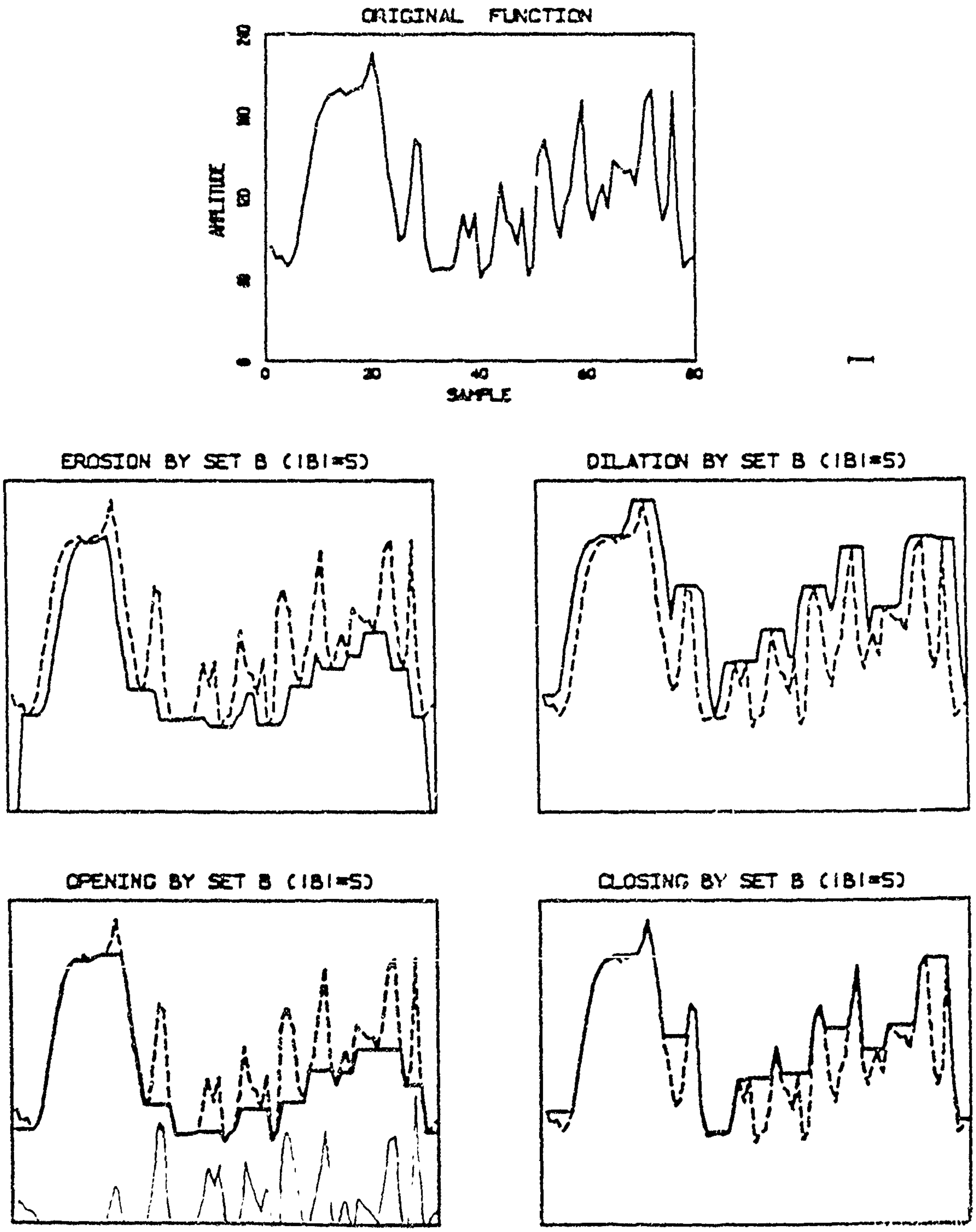




Morphological Signal Analysis

Top Hat Operator:

$$\mathcal{T}(f, B)(x) = f(x) - \mathcal{O}(f, nB)(x)$$



Systems for Thinning - III

Edge Detection:

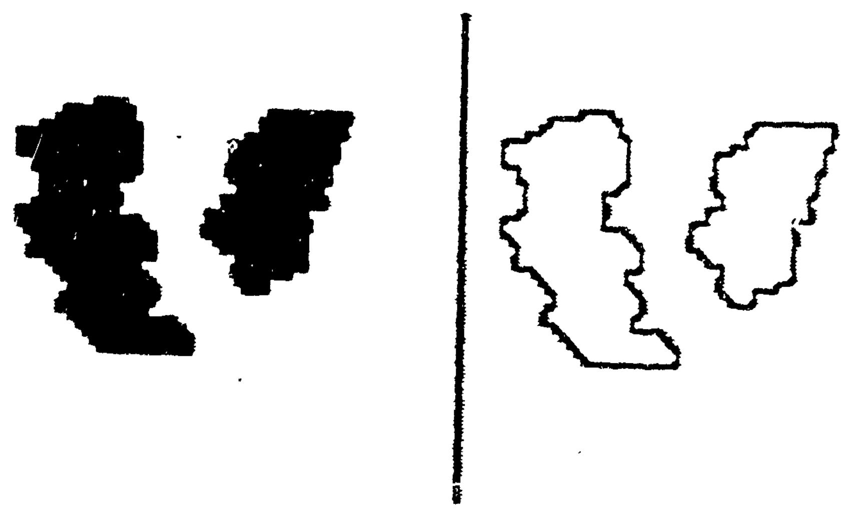
The edges of a binary image can be extracted using an operator of the form

$$\mathcal{F}(X, nB) = X / \mathcal{E}(X, nB)$$

where B is a small symmetric structuring element.

For greyscale images, the corresponding operator is

$$\mathcal{F}(f, nB) = f - \mathcal{E}(f, nB)$$



Rank-Order Systems - I

Consider a system with input $f(x)$ such that the output signal $\mathcal{R}_k(f, B)(x)$ is obtained by sorting the values of the input subset $\{f(z) : z \in B_x\}$ and assigning the kth number in the resulting list as the output value. (Assume N is the number of points in the set B.) Then

$$\begin{aligned}
\mathcal{R}_1(f, B)(x) &= \min_{z \in B_x}[f(z)] = \mathcal{E}(f, B)(x) \\
\mathcal{R}_N(f, B)(x) &= \max_{z \in B_x}[f(z)] = \mathcal{D}(f, B)(x) \\
\mathcal{R}_{(N+1)/2}(f, B(x) &= \text{median } [f(z) \text{ for } z \in B_x]
\end{aligned}$$

Rank-order systems are increasing and translation-invariant.

Rank-Order Systems - II

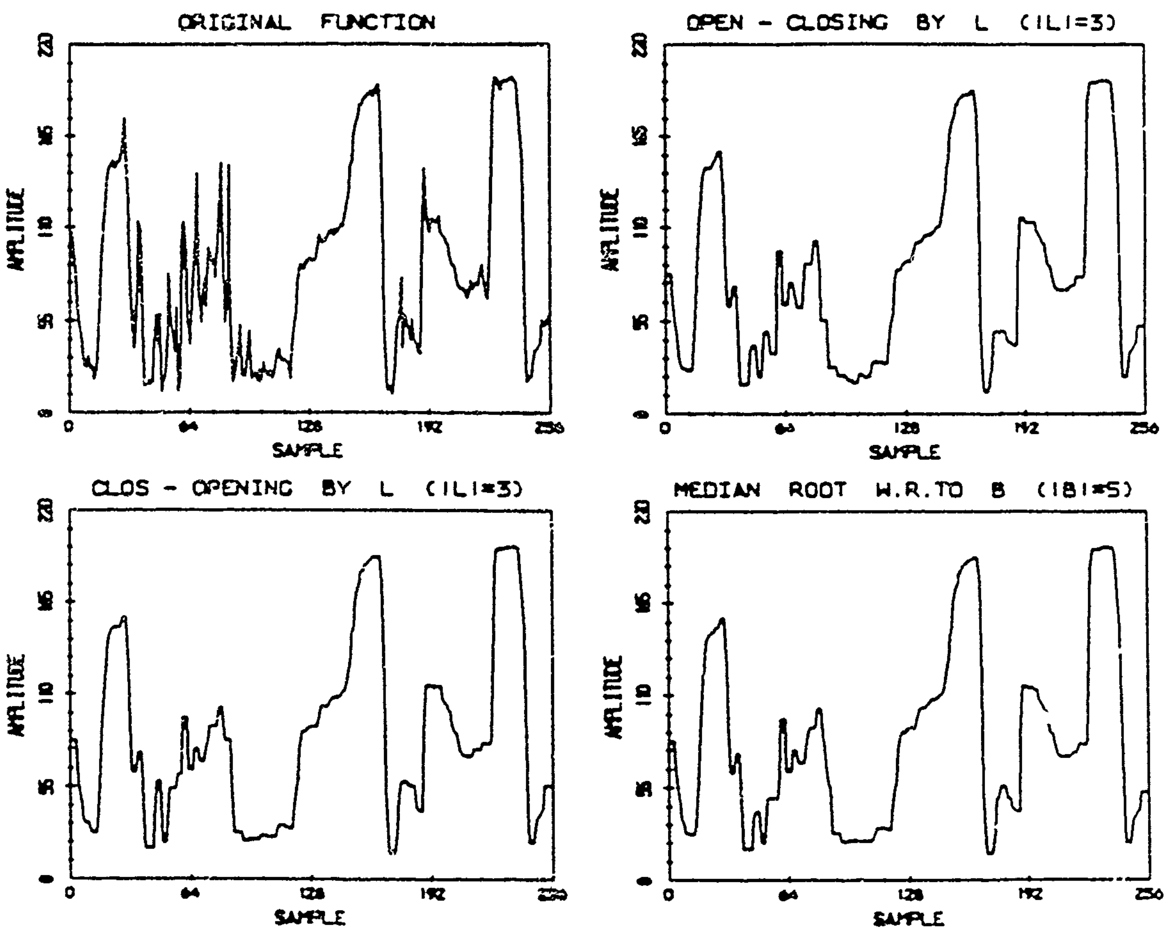
The kth rank-order system with structuring set B (window) is equivalent to a union of erosions by all the subsets of B containing k points. For example, consider a 3-point median filter.

$$\mathcal{M}_3(f, B)(x) = \text{median}\,[f(x-1), f(x), f(x+1)]$$

$$= \max \begin{bmatrix} \min[f(x-1), f(x)] \\ \min[f(x-1), f(x+1)] \\ \min[f(x), f(x+1)] \end{bmatrix}$$

Rank-Order Systems - III

Relations to other morphological systems:

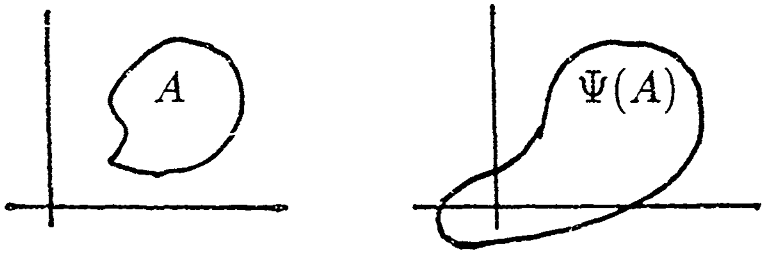


Kernel Representations - I

Consider an increasing translation-invariant system $\Psi(X)$. The *kernel* of this system is defined to be the collection of sets

$$\mathrm{Kern}(\Psi) = \{A : 0 \in \Psi(A)\}$$

where 0 is the origin.



Any increasing translation-invariant system can be exactly represented as a union of erosions by *all* its kernel elements; i.e..

$$\Psi(X) = \bigcup_{B \in \mathrm{Kern}(\Psi)} \mathcal{E}(X, B)$$

In general the kernel may contain an infinite number of elements, but sometimes it is possible to represent a system with only a finite subset (basis) of the kernel elements.

Kernel Representations - II

For function processing systems, the analogous result is

$$\psi(f,g)(x) = \max_{g \in \mathrm{Kern}(\psi)} [\mathcal{E}(f,g)(x)]$$

Recall the example of the previous 3-point median filter.

$$\mathcal{M}_3(f,B)(x) = \text{median } [f(x-1), f(x), f(x+1)]$$

$$= \max \begin{bmatrix} \min[f(x-1), f(x)] \\ \min[f(x-1), f(x+1)] \\ \min[f(x), f(x+1)] \end{bmatrix}$$

$$= \max_{B_i} [\mathcal{E}(f, B_i)(x)]$$

where $B = \{-1, 0, 1\}$, $B_1 = \{-1, 0\}$, $B_2 = \{-1, 1\}$, and $B_3 = \{0, 1\}$. Thus the kernel basis of the 3-point median filter is

$$\mathrm{Kern}(\mathcal{M}_3) = \{B_1, B_2, B_3\}$$

Kernel Representations - III

Which systems have finite kernels?

- Basic morphological systems – erosion, dilation, opening, and closing.
- Rank-order systems including median filters.
- An interesting class of linear shift-invariant systems.
- Wilcoxon filters – combination of median and linear filters (Crinon).
- Shape recognition window transformations (Crimmins and Brown).

Design of Morphological Systems

- Powerful theory – much structure.
- How do you specify a desired system?
- How do you synthesize a system meeting given specifications?

  * Can you find a kernel basis?

- How do you find the most efficient implementation for a given architecture?

Morphological Signal Processing Systems: Part II

- Application of morphology to FLIR images
- Concerns of computer aided morphology
- Introduction to LISP
- Numeric and symbolic processing of morphological expressions

Forward Looking Infra Red (FLIR) images are characterized by:

- compact light (concentrated heat) regions corresponding to man-made objects
- darker (cooler) background
- light distractions such as a forest or trees
- poor contrast
- no precise geometrical cues
- unknown object size

The top hat transformation is defined as:

$$f_{top_{nL}}(\vec{x}) = (f - \max\{f_{nL_0}, f_{nL_{90}}\})(\vec{x})$$

where n is an integer multiplier and L_0 and L_{90} are horizontal and vertical line structuring elements, respectively, centered at the origin and having overall length equal to 3, i.e., $||L_0|| = ||L_{90}|| = 3$.

To exploit the high contrast of the man-made object a simple approximation to the gradient is defined as:

$$\tilde{G}(\vec{x}) = (f \oplus B)(\vec{x}) - (f \ominus B)(\vec{x})$$

where a rhombus of size 1 was used for B.

The top hat transformation with:

$$\|nL_0\| = \|nL_{90}\| = 2n + 1$$

separates the compact areas, whose largest dimension (height or width) is less than $2n + 1$, from the larger non-compact areas such as a horizontal band or a forest.

The object location is found by taking the mean of the grey scale maximum locations of the gradient

$$\vec{x} = \frac{1}{M} \sum_{k=1}^{M} \vec{x_k}$$

where M is the number of times the maximum grey scale value is found, $\vec{x_k}$ is the location of the k^{th} maximum and $\vec{x}$ is the desired location. If there is only one maximum in the image then a point on the edge of the object is acceptable.

By using a slightly different top hat transformation of the form:

$$(f_{top_{nL}})(\vec{x}) = (f - \max\{f_{nL_0}, f_{nL_{90}}, f_{nL_{45}}, f_{nL_{135}}\})(\vec{x})$$

one may suppress some of the noise in the top hat images since this step more cleanly separates the object from the whole image, making the subsequent gradient processing less susceptible to false peaks.

The processing steps for the FLIR data were:

- Close with rhombus of size 1
- Sequence of Top Hat Transformations
- Find maximum of gradient
- Average maximum locations to find object

SUMMARY

- sequence of top hat transformations coupled with gradient processing able to locate single man-made object
- by changing the method of object location selection one can locate an arbitrary number of objects
- FLIR object detection can not be solved using solely these methods
- require additional information for more complete solution

Computer Aided Morphology - CAM

How does one select the sequences of operations to perform a task?

Serra suggests five rules for organizing sequences of morphological operations:

- Review possible modes for information reduction (i.e. reducing a structure)
- Order commutative processes so the largest simplification is made first
- Determine groupings of useful interconnected operations
- Minimize the effects of interaction of sequential operations
- Remember that it is possible to back-track to regain lost information

Requirements for computer aided morphology include:

- Representation of reference properties of operations
- Method of selecting operations based on assessment of properties
- Means of interpreting user's requirements
- Heuristic knowledge of operations and structuring elements

Introduction to List Programming
(Common LISP)

- Fundamental structures are word-like objects called atoms
- Groups of atoms form sentance-like objects called lists
- Atoms and lists, collectively, are called symbolic expressions
- Compound data objects and procedures are lists and can be used interchangably

Symbolic Expression Examples

Atoms may be numbers or symbols:

- 3.1415, ATOM, SYMMETRIC-STRUCTEL

Lists consist of a left parenthesis, followed by zero or more atoms or lists, followed by a right parenthesis:

- (THIS IS A (LIST)), (3.1415)

Expressions are typically lists whose first element is the name of a procedure to be evaluated, or executed:

- (+ (/ 4 2) (- 7 3)), (MAX POINT1 POINT2)

Built in features of LISP include:

- Numeric primitive operations such as $+, -, \times, \cdots$
- Floating point and integer arithmetic capabilities
- Symbolic primitives for list processing such as CAR, CDR
- Ability to evaluate arbitrary lists

Symbolic and Numeric Manipulations of Morphology Expressions

Features that a system for manipulating expressions should possess include:

- Numeric processing of expressions (i.e. computation of an erosion)
- Symbolic manipulation of an expression (i.e. simplification of sequence of operations)

In our work the symbolic system surrounds the numeric processing core.

The numeric manipulation of expressions requires:

- Natural signal representation (abstract data objects)

- Inquiry operations for extracting signal information

- Functional specification of useful operators (i.e. erosion, dilation, ...)

A representation of signals for numerical processing should be based on the following observations of signals:

- Signals are immutable
- Signals are identified and distinguished by their region of support and sample values.
- Signals are organized into signal classes
- Signals exhibit deferred evaluation

By considering a sine wave to be a function of three variables (ω, φ, and N) we can define the class of sinusoids by:

$$x_{\omega,\varphi,N}[n] = \sin(\omega \cdot n + \varphi) : n = 1, \ldots, N$$

Any particular sinusoid may be formed by binding parameters with actual values:

Setting $\omega = \frac{2\pi}{7}$, $\varphi = \frac{\pi}{4}$, and $N = 10$ generates

$$\sin(\frac{2\pi}{7} \cdot n + \frac{\pi}{4}) : n = 1, \ldots, 10$$

The notion of signal classes naturally leads to subclasses, generating a hierarchy of signal classes.

As an example, zero-phase sinusoids form a subclass of sinusoids:

$$x_{\omega,N}[n] = \sin(\omega \cdot n) : n = 1, \ldots, N$$

Specification of a signal class requires:

- Name for the class (i.e. sine-wave)
- Parameters for distinguishing specific signal instances
- Functional specification for computing samples
- Parent type for property inheritance
- A signal finder for creating specific signals

Annotated terminal session defining the CONSTANT signal class:

```
(1) :  (defsigtype constant  :parameters (length sample-value)
                             :a-kind-of  basic-signal
                             :finder     signal-constant
                             :init       (dimension length)
                             :fetch      ((n) sample-value) )

 ==>  CONSTANT               ; returns the class name

(2) :  (signal-constant 10 1)  ; create a signal instance

 ==>  SIGNAL-1                ; returns unique name

(3) :  (signal-fetch signal-1 3)  ; request sample at n = 3

  ==>  1                          ; value of sample at n = 3

(4) :  (signal-what signal-1)     ; inquire about signal-1

  ==>  (signal-constant 10 1)     ; returns the definition

(5) :  (signal-constant 10 1)     ; re-specify the signal

  ==>  SIGNAL-1                 ; signal name is unique
```

Symbolic Manipulations

Rule based systems provide a flexible mechanism for representing morphological knowledge for:

- Expression simplification
- Generation of equivalent forms

Sample manipulation rules

```
(Open-Idempotence        ; Rule name
      (:FORM             ; If the input matches this form
             (OPEN (OPEN ?IMAGE ?SE1) ?SE2) )
      (:TEST             ;    and satisfies this condition
             (SE1 OPEN WR/T SE2) )
      (:RESULT           ; Then replace input with result
             (OPEN IMAGE SE1) ) )


(Erosion-of-structel-union
    (:FORM          ; The input form
          (ERODE ?IMAGE (UNION ?SE1 ?SE2)) )

    (:RESULT        ; The equivalent output form
          (INTERSECT (ERODE IMAGE SE1) (ERODE IMAGE SE2)) )
)
```

Simplification Example

```
(1) :    (SIMPLIFY '(CLOSE
                         (OPEN
                              (OPEN SIGNAL-1 LIN000)
                              VEC000 )
                         RHOMBUS ) )

  ==>    (FOUND RULE - OPEN-IDEMPOTENCE)

; System has found a rule and applied it

  ==>    (CLOSE (OPEN SIGNAL-1 LIN000) RHOMBUS)

; Evaluate the expression

(2) :    (EVAL '(CLOSE (OPEN SIGNAL-1 LIN000)))

; Return the resulting signal object

  ==>    SIGNAL-3
```

Part of ARO 26131.1-EL-CF

APPLICATIONS OF MORPHOLOGY IN INDUSTRY

STEPHEN S. WILSON

APPLIED INTELLIGENT SYSTEMS, INC.
110 PARKLAND PLAZA
ANN ARBOR, MI 48103

MATHEMATICAL MORPHOLOGY

IS A MATHEMATICAL MODEL ADDRESSING THE NEED TO ANALYZE PICTURES TO GAIN KNOWLEDGE.

IT IS A SET OF TOOLS. FOR A PARTICULAR PROBLEM, THE METHOD WILL SELDOM TELL YOU EXACTLY WHAT TO DO, OR HOW TO DO IT - THE USER MUST GAIN AN INTUITION FOR THE STRENGTHS AND WEAKNESSES OF THE METHOD, AND DECIDE FOR HIMSELF WHICH TOOLS TO USE.

1.0

THE FIRST OBJECTIVE OF THIS COURSE IS TO FOCUS ON THE VARIOUS TOOLS FROM AN INTUITIVE POINT OF VIEW. A RIGOROUS MATHEMATICAL INSIGHT IS OPTIONAL IN USING THESE TOOLS, BUT CAN COME LATER BY STUDYING THE LITERATURE ON THE SUBJECT. A KNOWLEDGE OF SET THEORY AND BOOLEAN ALGEBRA WOULD BE REQUIRED.

A SECOND OBJECTIVE IS TO UNDERSTAND HOW THE TOOLS CAN BE APPLIED TO VARIOUS CATEGORIES OF APPLICATIONS - FOR SPECIFIC APPLICATIONS, WHICH TOOLS WORK WELL, AND WHICH DO NOT.

1.0

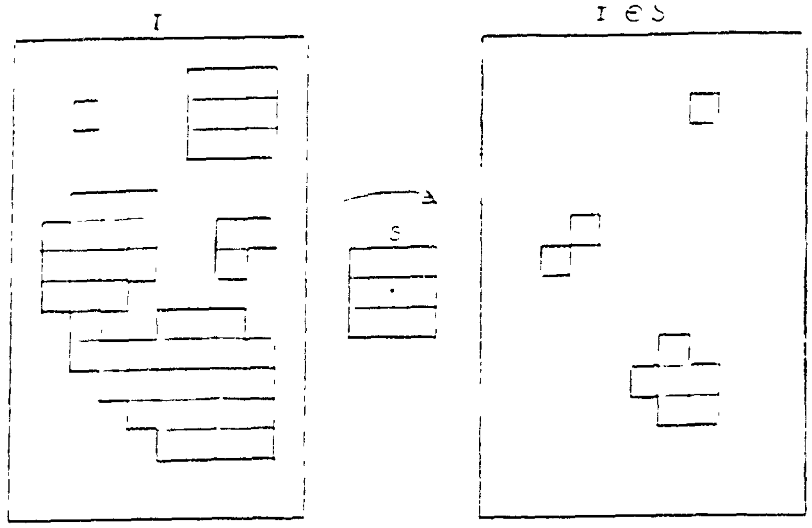
THERE ARE SOME APPLICATIONS WHERE THE TOOLS OF MATHEMATICAL MORPHOLOGY ARE NOT APPROPRIATE. WE WILL FOCUS ON THOSE APPLICATIONS THAT DO WORK.

MORPHOLOGY, AT ITS BEST IS MIXED WITH OTHER TECHNIQUES. THESE OTHER TECHNIQUES WILL BE DISCUSSED WHEN THEY HAVE RELEVANCE TO THE PRINCIPLES OF MORPHOLOGY.

BLANK PAGES

PAGES 3-7

EROSIONS



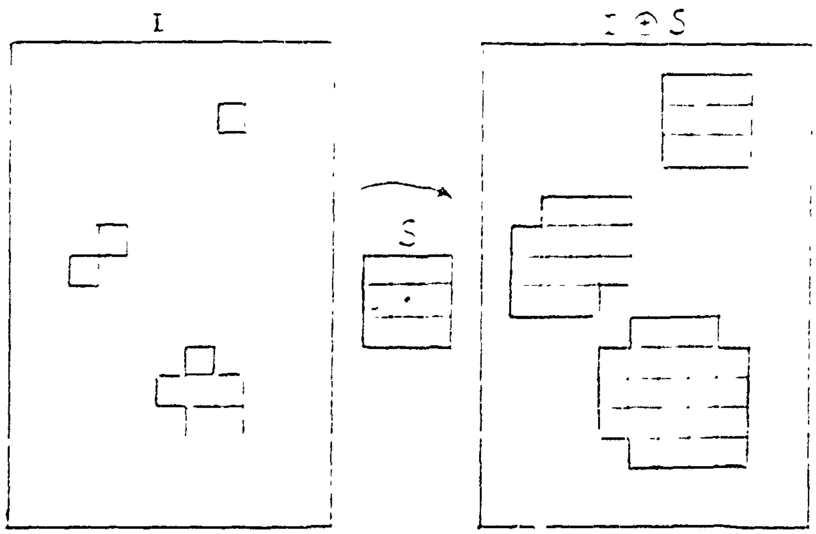
I = image S = structuring element

Erosion: $I \ominus S = \bigcap_{s_i \in S} I_{-s_i}$

note: subscript means translate

Image I eroded by structuring element S means wherever S fits in I, mark the center of S on I.

DILATIONS

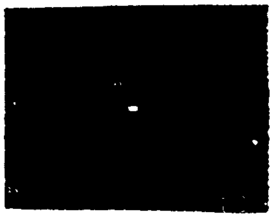


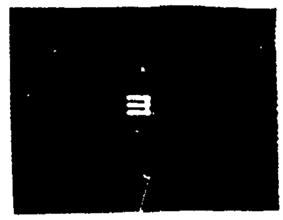
Dilation $I \oplus S = \bigcup_{s_i \in S} I_{s_i}$

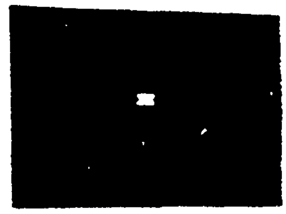
Wherever the center of $\hat{S}$ hits I, mark $\hat{S}$ on I or wherever S touches I, mark the center of S on I.

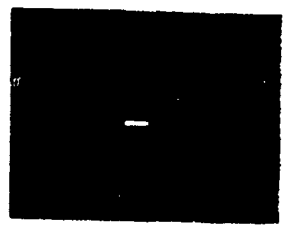
BLANK PAGE

DILATION EXAMPLES
FICTITIOUS PROGRAM

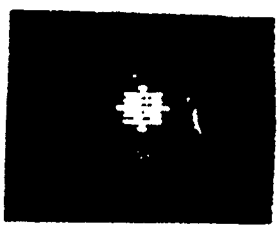


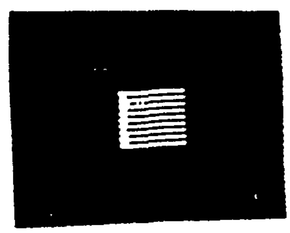


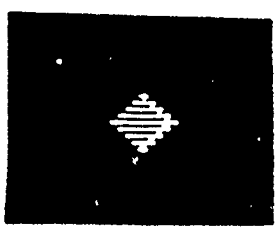


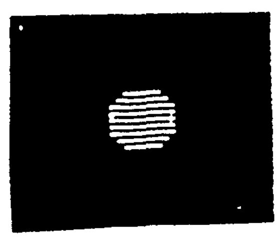


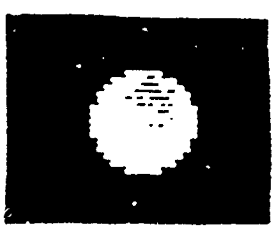
```
SYSTEM-INIT
DOT
LOOK
DOT
D*
LOOK
DOT
D+
LOOK
DOT
D.
LOOK
DOT
D|
LOOK
DOT
D-
LOOK
DOT
D-0
LOOK
DOT
D*  D*  D*  D*
LOOK
DOT
D+  D+  D+  D+
LOOK
DOT
D+  D+  D*  D*
LOOK
DOT
D+  D+  D*  D*
D+  D-0
LOOK
DOT
D+  D*  D|  D|
LOOK
END
```

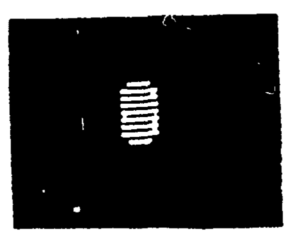






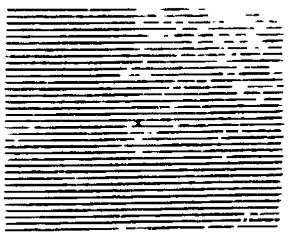


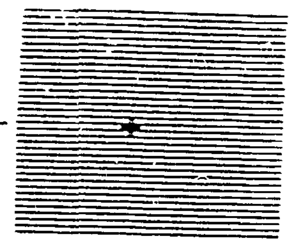


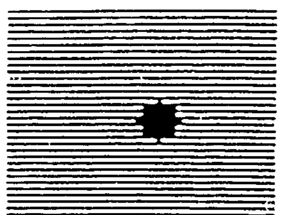


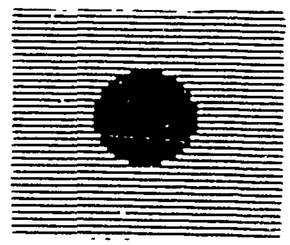
BLANK PAGE

EROSION EXAMPLES
FICTITIOUS PROGRAM

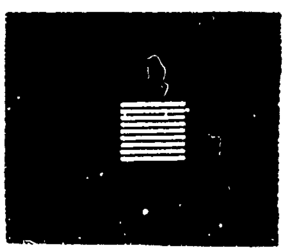


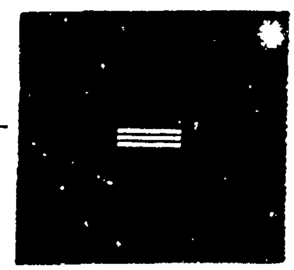


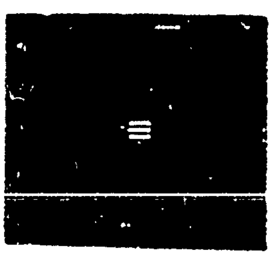


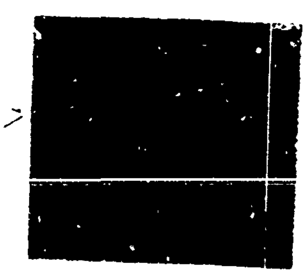


```
SYSTEM-INIT
BLACK-DOT
LOOK
E+
LOOK
BLACK-DOT
E-O
LOOK
BLACK-DOT
E+   E*   E+   E*
E+   E-O
LOOK

BOX
LOOK
E|   E|   E|
LOOK
BOX
E+   E+   E+
LOOK
E+   E+   E+   E+
E+   E+
LOOK
END
```



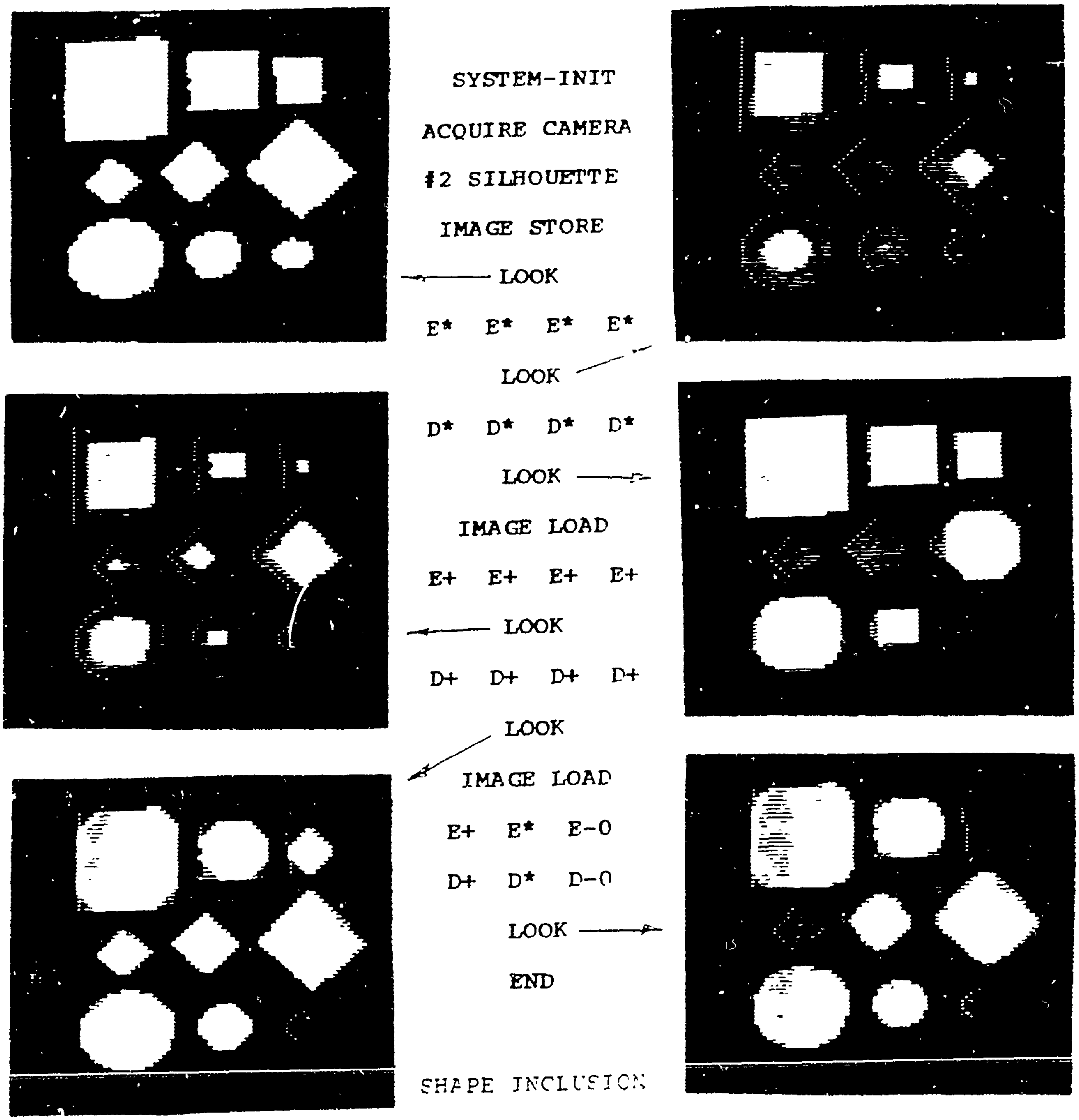




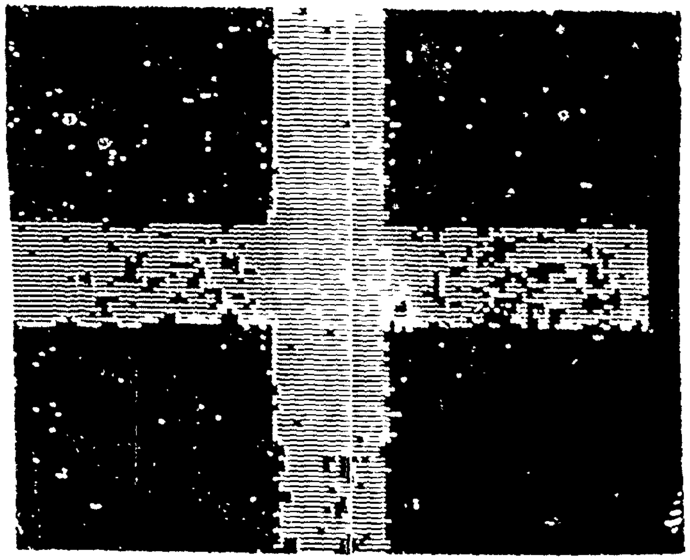


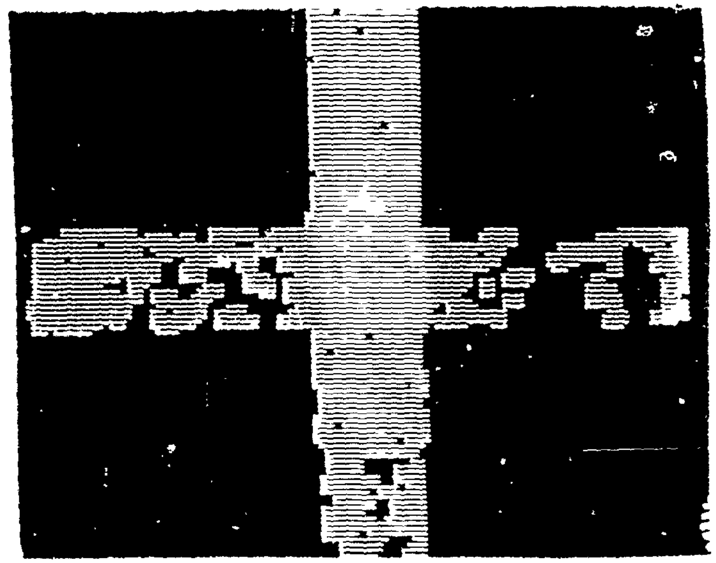
BLANK PAGE

OPENING OR SHAPE INCLUSION EXAMPLES



SHAPE INCLUSION





NOISE FILTERING USING
OPENINGS AND CLOSINGS

SYSTEM-INIT

CAMERA

#2 SILHOUETTE

LOOK

IMAGE STORE

E* D* ...SALT

LOOK

IMAGE-LOAD

D* E* ... PEPPER

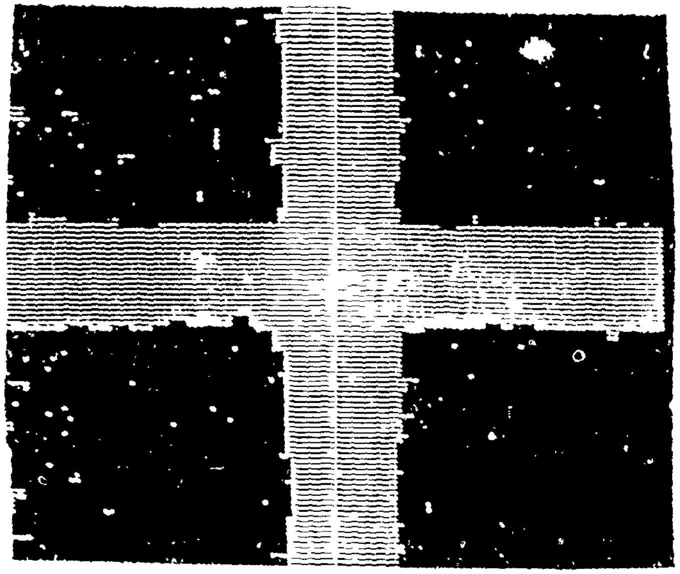
LOOK

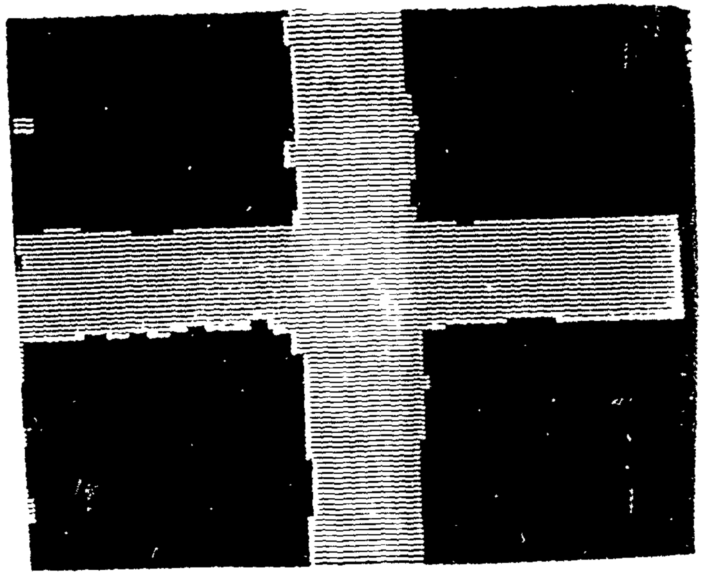
IMAGE LOAD

D* E* E* D*

...SALT AND PEPPER

LOOK





OTHER OPERATIONS
- Hit or miss (Serra 1982)

Two structuring elements:

$$\left.\begin{array}{l} H = \text{"Hit"} \\ M = \text{"Miss"} \end{array}\right\} S$$

$I \circledast S = \{ i : H \subset I;\ M \subset I^c \}$

i.e. the transformed pixel is "1"
if H is included in the object foreground
and M is included in the object background.

Thus, erosions only use AND operations,
dilations only use OR operations,
hit or miss uses both operations.

There are many important examples.

TOFOLOGICAL FILTERS

A "FLUG" will cause a kind of dilation, but it will dilate preferentially into areas for which white is concave on black. These figures show the effect of PLUG on swiss cheese: the holes are filled up while the outer dimensions of the cheese stays the same.

Formula: $PLUG(I) = C \text{ OR } (\text{AND}_{neigh} \; N)$

SYSTEM-INIT

AQUIRE-CAMERA

SILHOUETTE

LOOK

PLUG

PLUG

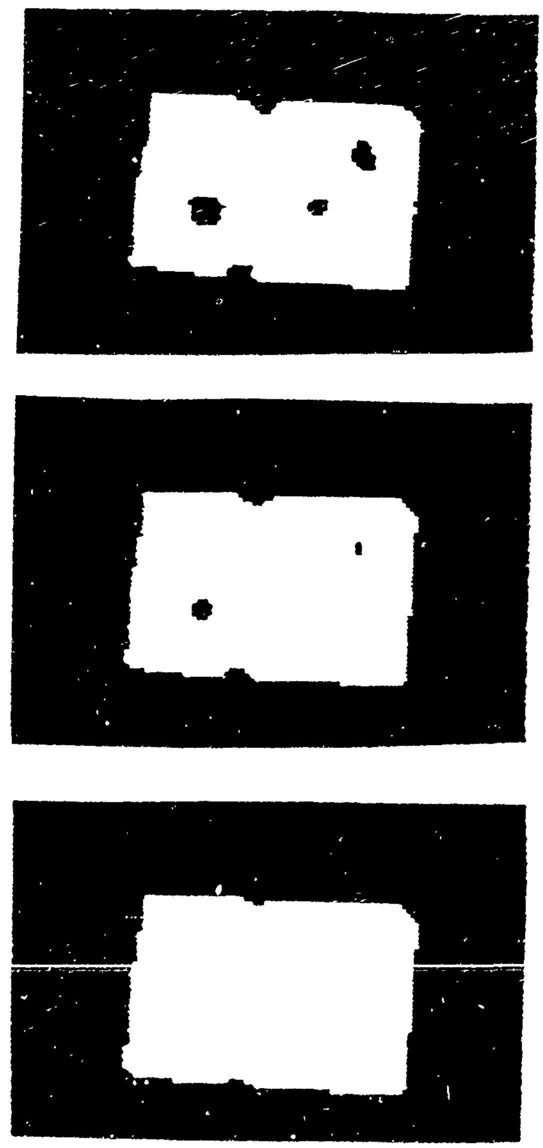
LOOK

PLUG

FLUG

LOOK

END

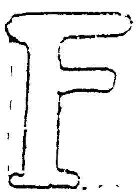


CONVEX HULL

Put a rubber-band around the object.

CONVEX HULL SIMPLIFIED

The plug operation repeated enough times will put a box around the object.

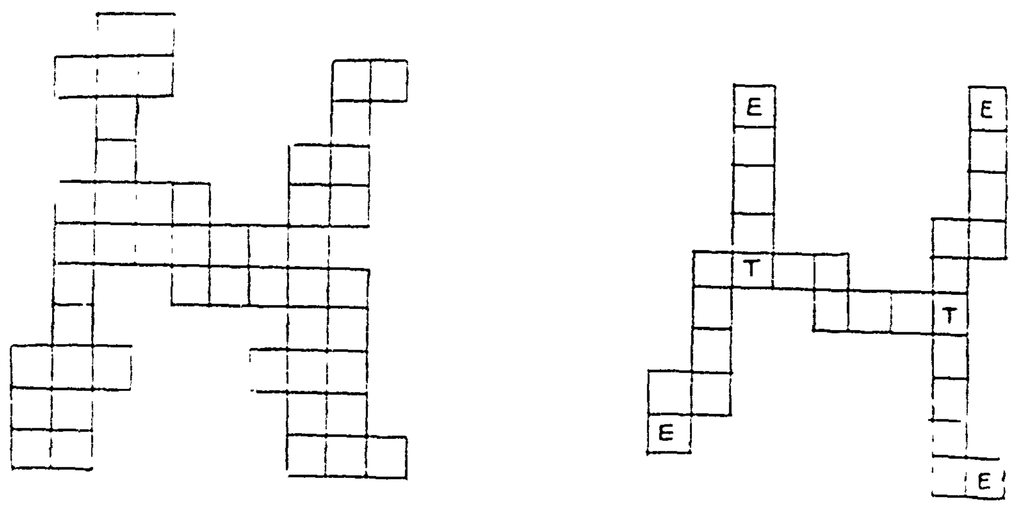


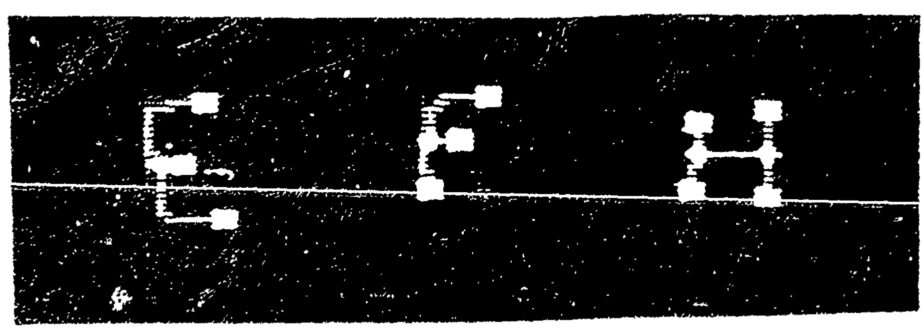
SKELETONIZE OR THINNING

A type of erosion except the last dot or thinnest line does not erode away completely.
(i.e. maintain the connectivity)

Features such as end points and tee connections become apparent.

Cannot be done in one 3x3 structuring element. Must use 4 elements in succession: thin N, S, E, and W. Must be repeated depending on the thickness of the object.

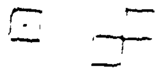




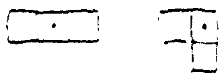
FEATURE FINDING FUNCTIONS

Blank out image except where there is the specified feature. Useful after skeletonizing operations.

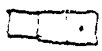
Find dot - there is an isolated pixel.

yes→  no→ 

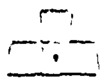
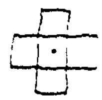
Find line - the middle of a line segment.

yes→  no→ 

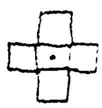
Find end point - of a line segment.

yes→  no→ 

Find tee connection.

yes→  no→ 

Find four connection.

yes→  no→ 

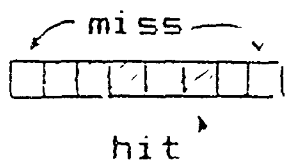
More complex finding functions can be combinations of the above.
e.g. find line OR end point.

COVARIANCE

This is a large field. We will not go into details in this course. Need a knowledge of statistics.

Examples.

Chords of size N.



Set covariance of size N

Use various angles and sizes. Tally results.
Look at statistics.
Good for classifying textures such as in polished mineral cross-sections, or wood grain.

CONDITIONAL DILATION

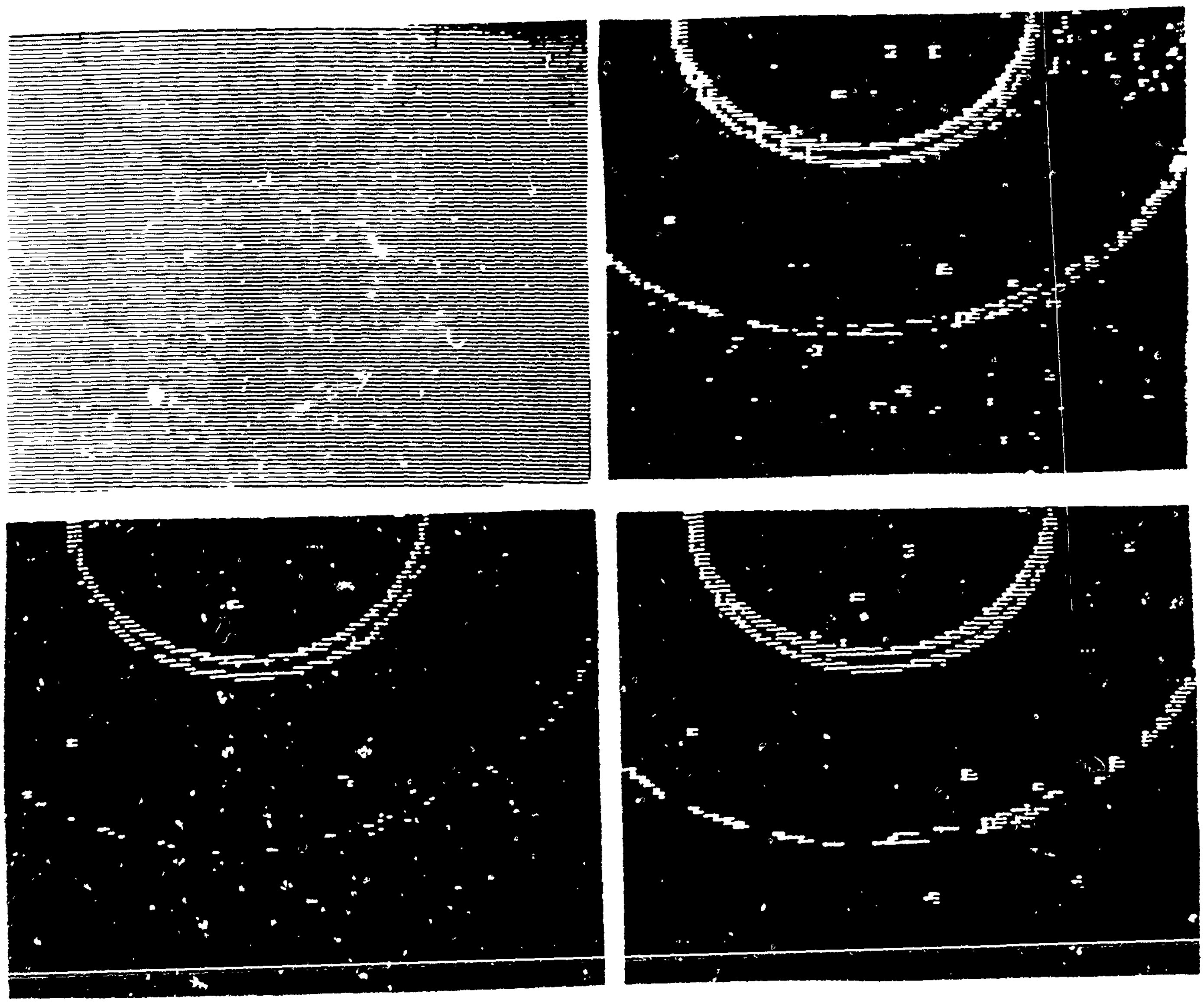
Two bitplanes are involved:

A "mask" or condition plane which will not be altered.

An "object" plane where pixels are to be dilated.

The object is generally smaller than the mask.

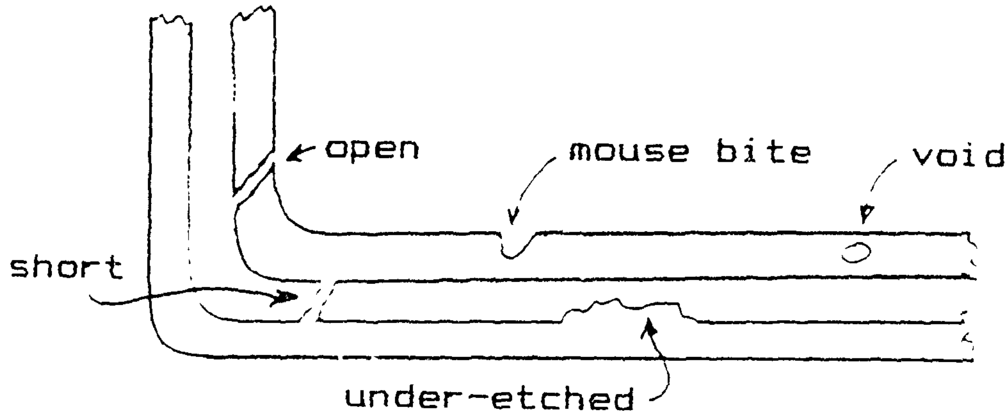
The transformation: dilate the object plane under the condition that it does not grow outside the bounds of the mask.



BLANK PAGE

BLANK PAGE

PRINTED CIRCUIT BOARD INSPECTION



Inspection by rule.
To look at lead quality, open with a small disk.

To look at spacing between leads, close with a small disk.

Then shrink.

Then do feature finding.

DIRECTIONAL FILTERING

Problem: Thresholding after edge detection of a low contrast noisy picture results in a noisy binary image. Raising the threshold causes a loss of the image along with intended loss of noise.

Morphological filtering by opening or closing fails because of the noise density.

An effective solution:

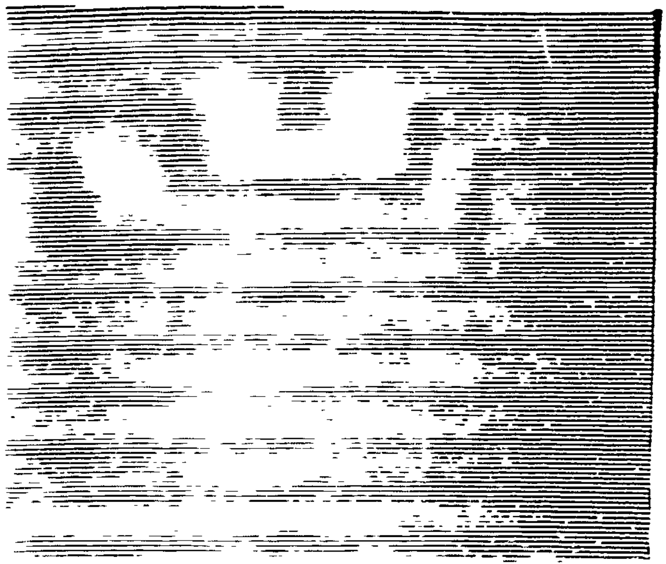
Create separate bit planes for the eight different edge directions.

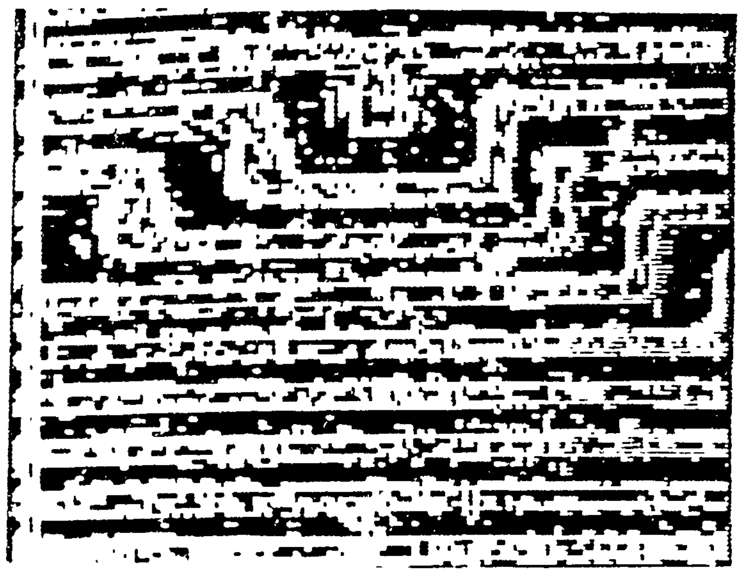
Threshold them separately with the same threshold.
The density of pixels around edges remains constant for the corresponding direction plane.
The density of random noise drops by a factor of eight so that morphological filtering is now more effective.

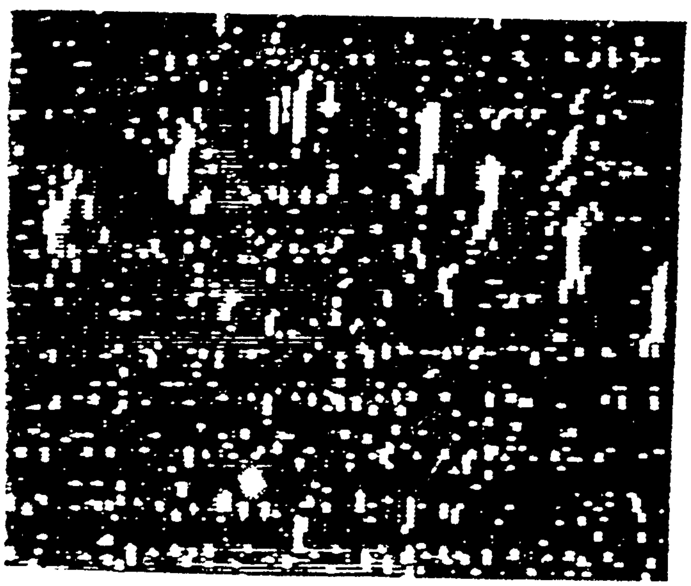
Filter each direction plane separately.

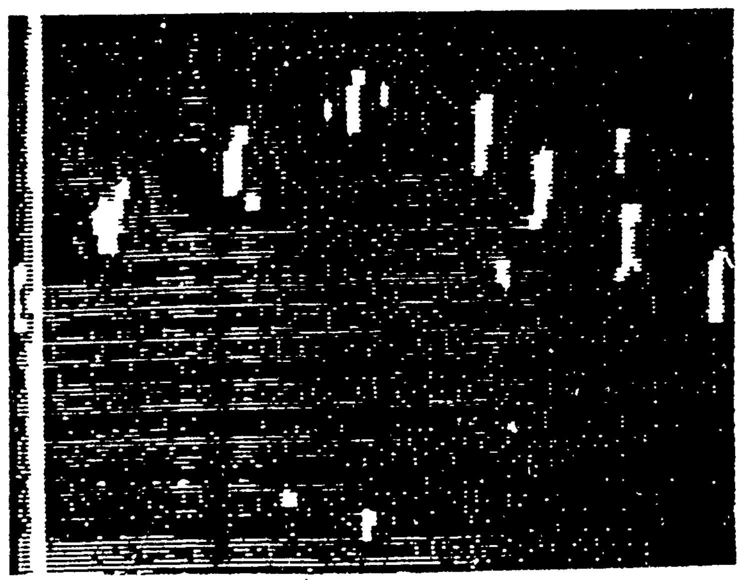
Combine into one plane if desired by an OR of the eight direction planes.

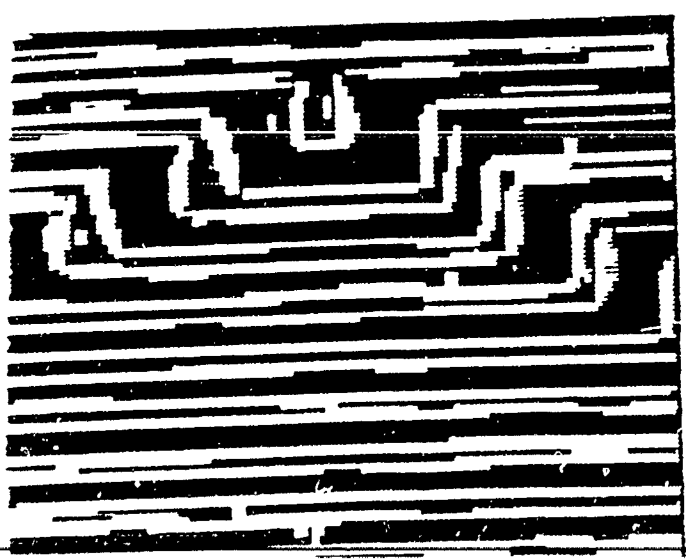
DIRECTIONAL FILTERING EXAMPLES











BLANK PAGE

FUZZY LOGIC

First, Aristotelian logic:
Two values - true or false = 1 or 0.

A	B	AND	OR
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

A	NOT A
0	1
1	0

Truth tables

Fuzzy logic - continuous values 0 → 1

0 ⇒ False
.2 ⇒ I doubt it
.5 ⇒ Maybe, maybe not
.8 ⇒ I think so
1. ⇒ You're darn right

Operation		Replacement
A AND B	→	MINIMUM(A,B)
A OR B	→	MAXIMUM(A,B)
$\overline{A}$ = NOT A	→	1 - A

De Morgan's rule still works

Binary: $\overline{A}$ AND $\overline{B}$ = $\overline{\text{A OR B}}$
Fuzzy: MIN(1-A,1-B) = 1 - MAX(A,B)

All other common theorems in boolean logic also hold with the above replacements.

FUZZY LOGIC APPLIED TO MORPHOLOGY

The grey level intensity is a fuzzy logic state.

Binary morphology operations have a direct fuzzy counterpart.

Dilate:	OR(nbhd)	$\longrightarrow$	MAX(nbhd)
Erode:	AND(nbhd)	$\longrightarrow$	MIN(nbhd)
Plug:	cen OR AND(nbhd)	$\longrightarrow$	MAX(cen,MIN(nbhd))

What is a "fuzzy threshold"?

3.1

IMPORTANT ISOMORPHISM

grey image $\xrightarrow{\text{fuzzy ops}}$ grey image $\xrightarrow{\text{threshold}}$ binary image

grey image $\xrightarrow{\text{threshold}}$ binary image $\xrightarrow{\text{binary ops}}$ binary image

The resulting binary image is the same in both cases!

Fuzzy logic is useful
when the threshold is adaptive or uncertain;
when combined with arithmetic operations.

BLANK PAGES

32-34

4.0

BEYOND MORPHOLOGY

The following operations strictly do not belong in a discussion of morphology. However, some principles are related, and they are often necessary to use along with morphology in order to successfully develop solutions to some image processing problems.

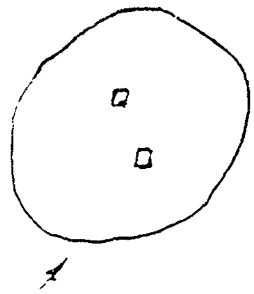
4.1

IMAGE ALGEBRA

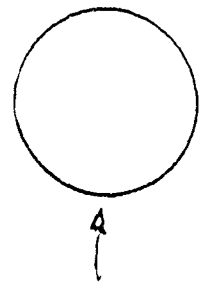
Supported by the Air Force and DARPA.
Under development at the University of Florida.
The objective is to be able to express all grey level image-to-image transformations.

MAJORITY VOTING LOGIC

Similar to 2-D erosions.



Object with
pixel dropouts



Structuring element
will not fit.

This object will not be detected with an opening.

Majority vote:
Allow a 95% fit of the structuring element to vote for a hit.

Noise dropouts can then be tolerated.

The percentage vote is a variable to be adjusted to give good performance.

A 100% vote is equivalent to an erosion.

CONVOLUTION
A DEFINITION

KERNEL

W1	W2	W3
W4	W5	W6
W7	W8	W9

INPUT IMAGE

		P1	P2	P3		
		P4	P5	P6		
		P7	P8	P9		

OUTPUT IMAGE

			C			

$$
\begin{aligned}
C = {} & W1 \times P1 + W2 \times P2 + W3 \times P3 \\
& W4 \times P4 + W5 \times P5 + W6 \times P6 \\
& W7 \times P7 + W8 \times P8 + W9 \times P9
\end{aligned}
$$

MOVE THE KERNEL WINDOW OVER THE ENTIRE IMAGE AND COMPUTE THE SUM OF PRODUCTS FOR EACH PIXEL.

SOBOL OPERATOR

X KERNEL

−1	Ø	1
−2	Ø	2
−1	Ø	1

Y KERNEL

1	2	1
Ø	Ø	Ø
−1	−2	−1

$$\text{SOBOL MAGNITUDE} = \sqrt{Cx^2 + Cy^2} \approx |Cx| + |Cy|$$

$$\text{SOBOL DIRECTION} = \text{ARCTAN } Cx/Cy$$

IMAGE	2	3	3	23	23	22
EXAMPLE	3	4	4	22	23	22
	4	3	3	22	21	21

Cx	1	79	77	2
Cy	−3	Ø	2	6
Cx + Cy	4	79	79	8

THE SOBOL OPERATOR IS AN EDGE DETECTOR.

BLANK PAGES

LAPLACIAN OPERATOR

KERNEL

-1	-1	-1	
-1	8	-1	÷ 8
-1	-1	-1	

IMAGE EXAMPLE	2	3	2	23	23	22
	3	4	4	22	23	22
	4	3	3	22	21	21
LAPLACIAN		1	-6.3	6.9	1	

A DIRECTION INDEPENDANT EDGE DETECTOR.
IT DETECTS SECOND DERIVATIVES ONLY -
A CONSTANT SLOPE GIVES ZERO OUTPUT.
IT IS A SIMPLE, ONE-PASS OPERATOR, BUT
IT IS NOT VERY SENSITIVE TO EDGES.

LARGE AREA AVERAGE

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \div 49$$

THE IMAGE WILL BECOME BLURRED.

SUBTRACT THIS FROM THE ORIGIONAL IMAGE FOR A HIGH PASS FILTER. THIS IS AN EXTENSION OF THE LAPLACIAN FILTER.

BLANK PAGE

GAUSSIAN CONVOLUTION

Ø	Ø	1	1	2	1	1	Ø	Ø
Ø	1	4	8	11	8	4	1	Ø
1	4	14	29	37	29	14	4	1
1	8	29	61	77	61	29	8	1
2	11	37	78	99	78	37	11	2
1	8	29	61	77	61	29	8	1
1	4	14	29	37	29	14	4	1
Ø	1	4	8	11	8	4	1	Ø
Ø	Ø	1	1	2	1	1	Ø	Ø

THIS IS AN EXAMPLE OF A GAUSIAN WHICH CAN OCCUR WITH DIFFERENT RADII.

THE GAUSSIAN BLURS THE IMAGE. MANY THINGS IN NATURE ARE BLURRED BY A GAUSSIAN.

BLANK PAGES

VECTOR CORRELATION

VECTOR KERNEL

(Ø,1)	(Ø,1)	(Ø,1)		
			(7,7)	
				(1,Ø)
				(1,Ø)

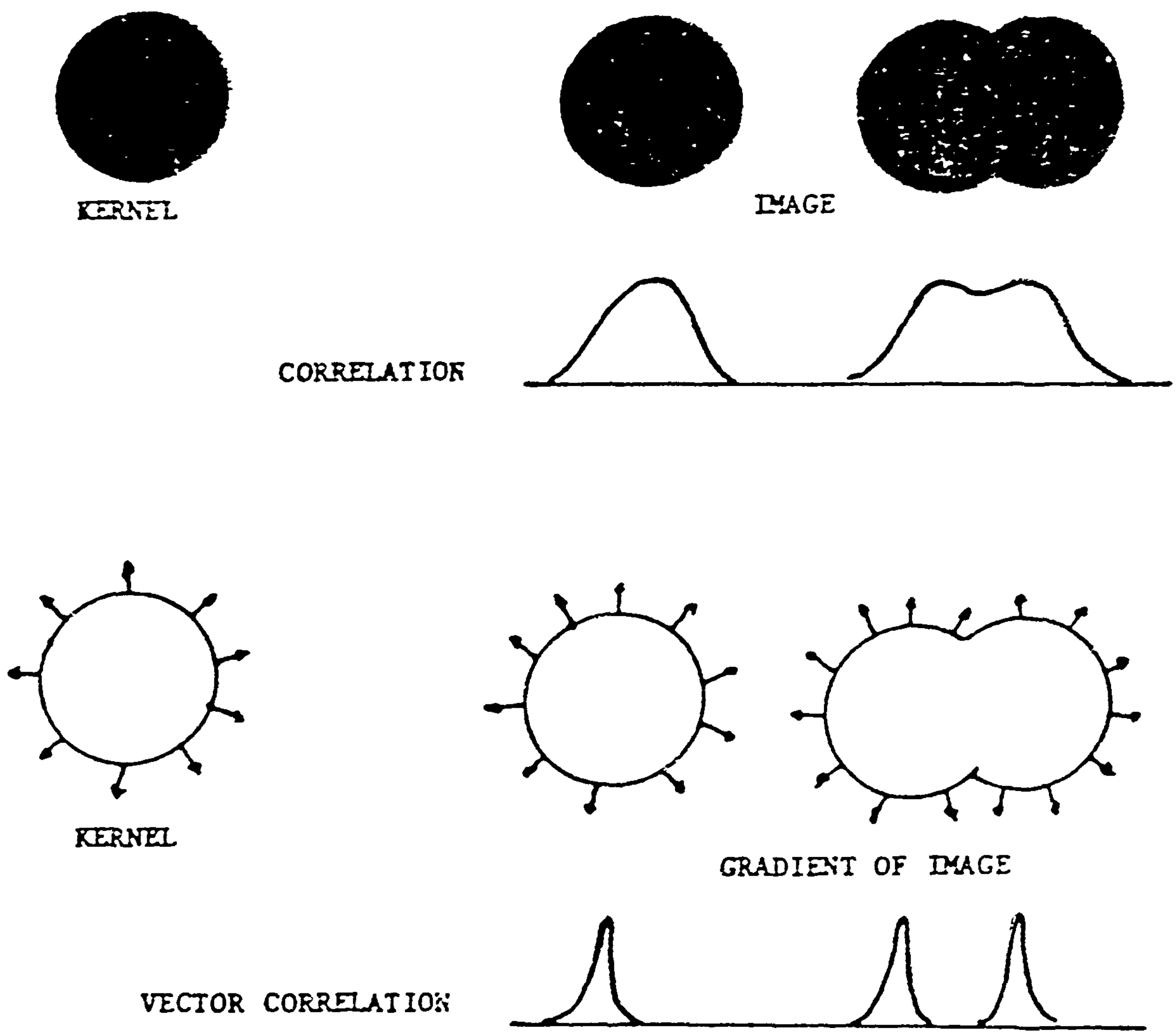
THE KERNEL $(K_x, K_y) = \vec{K}$ IS A VECTOR WITH MAGNITUDE AND DIRECTION.

THE IMAGE IS TRANSFORMED TO A VECTOR WITH MAGNITUDE AND DIRECTION USING, FOR EXAMPLE, THE SOBOL X AND Y KERNELS.

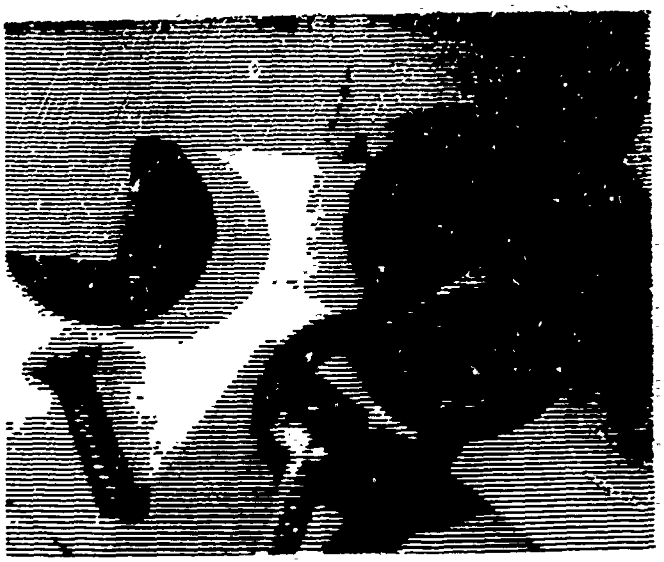
THE CORRELATION IS $C = \text{SUM OF } \vec{K_i} \cdot \vec{P_i}$

WHERE $\vec{K} \cdot \vec{P} = K_x \cdot P_x + K_y \cdot P_y$

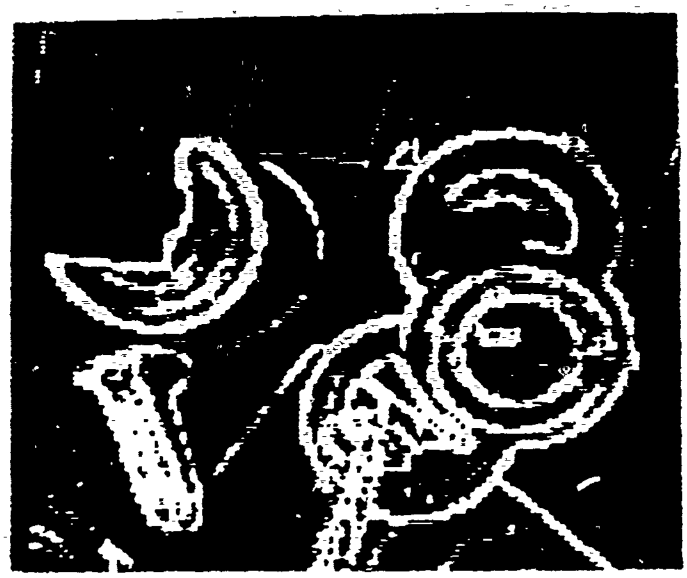
VECTOR CORRELATION IS A VERY ROBUST METHOD FOR FINDING EDGES. IN PRACTICE, APPROXIMATIONS TO THE ABOVE EQUATIONS ARE USED.


KERNEL
IMAGE
CORRELATION
KERNEL
GRADIENT OF IMAGE
VECTOR CORRELATION

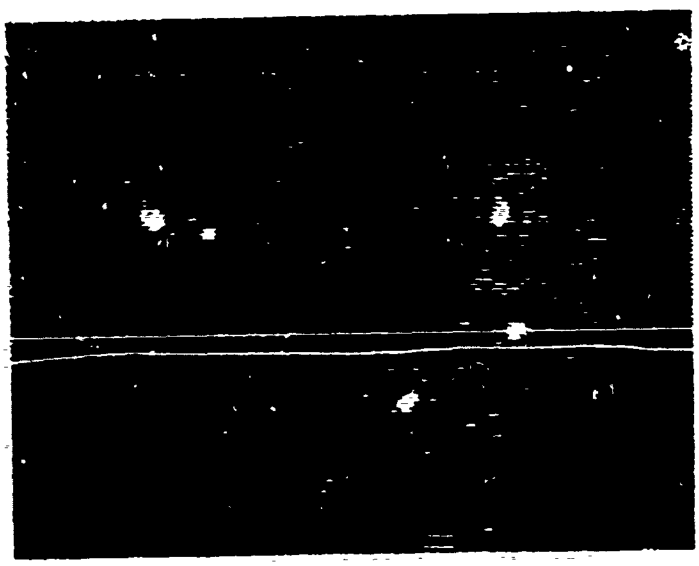
VECTOR CORRELATION EXAMPLE



Image

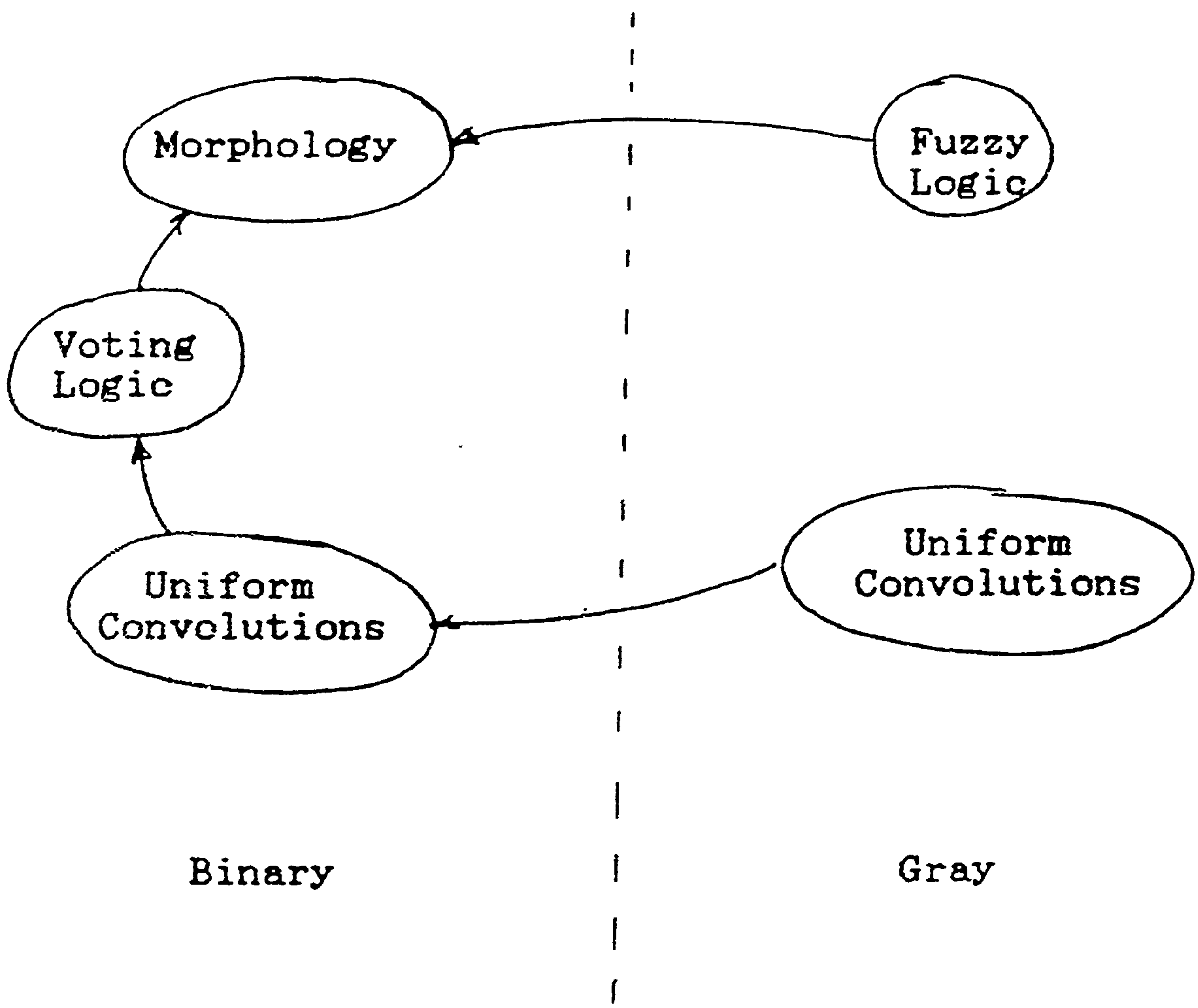


Gradient with Eight Directions



Vector correlation

RELATED CONCEPTS



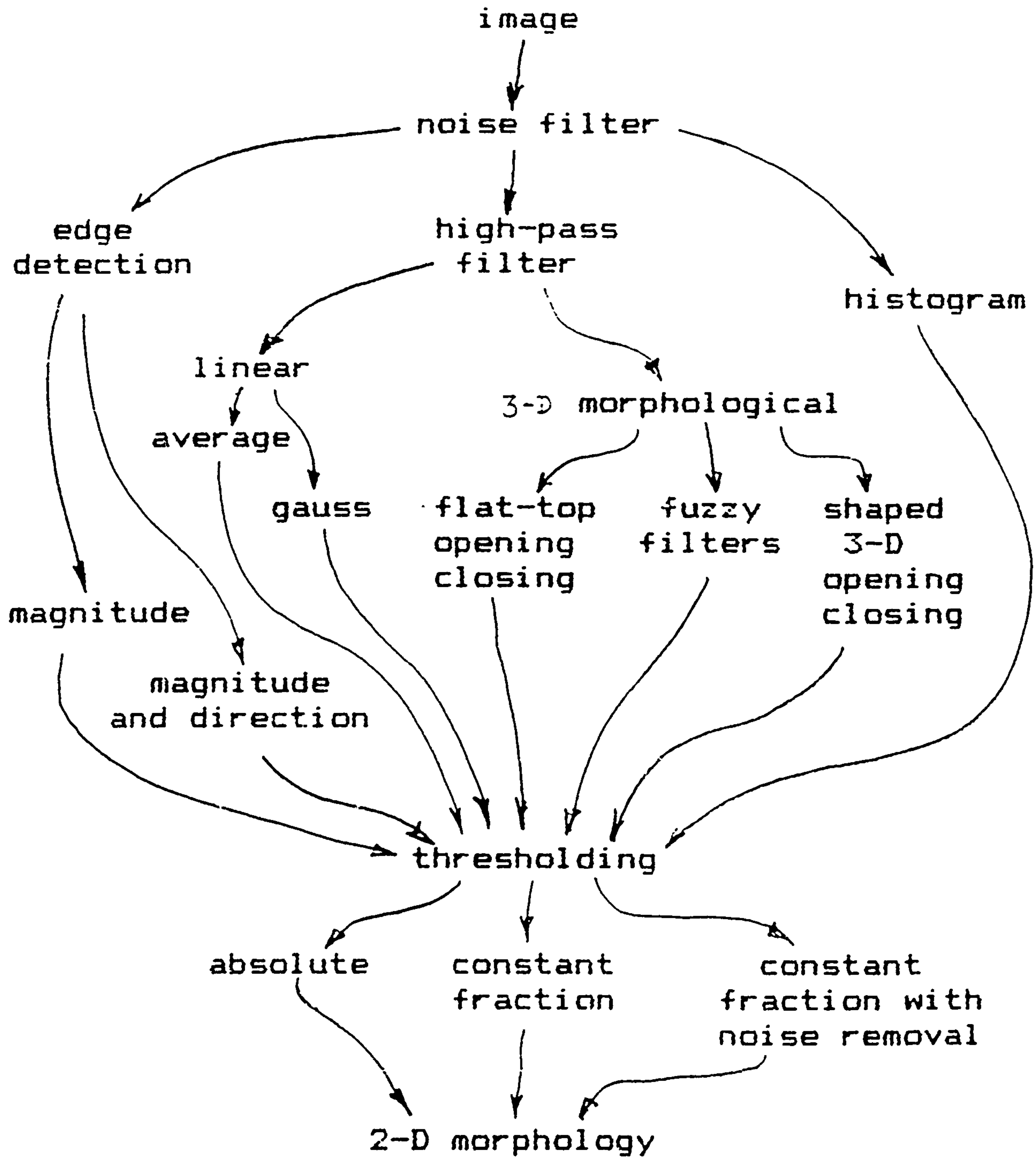
OPERATION HIERARCHY

General Correlation	$C = \sum_{shape} T*P$
Uniform Correlation	$UC = \sum_{shape} 1*P \quad P = 0 - 255$
Binary Uniform Correlation	$BUC = \sum_{shape} 1*P \quad P = 0 \text{ or } 1$
Voting Logic (Thresholded BUC)	$TBUC = \begin{cases} 1 \text{ if } BUC > T \\ 0 \quad \text{otherwise} \end{cases}$
Morphology (Maximum TBUC)	$MTBUC = \begin{cases} 1 \text{ if } BUC = max \\ 0 \text{ otherwise} \end{cases}$
Fuzzy logic	$? \quad = \quad MAX(P \text{ in shape})$

BLANK PAGES

52 - 61

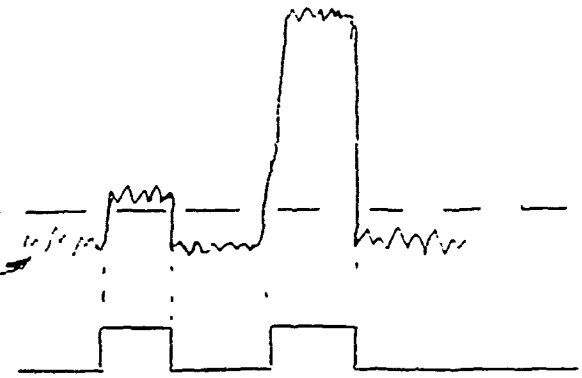
PROCESSING TECHNIQUES USING MORPHOLOGY
OR OTHER RELATED OPERATIONS



BLANK PAGES

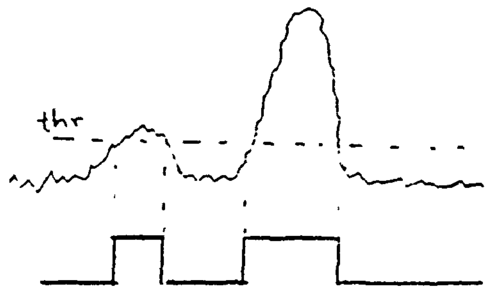
63-68

THRESHOLDS
FIXED THRESHOLD.

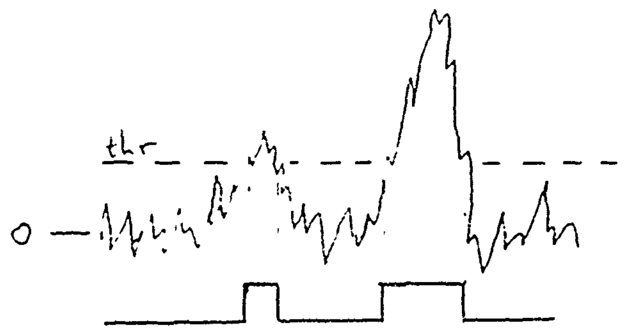


Set threshold
just above noise

A blurred image gives
an illumination
dependent width.



Non-linear filtering would help by allowing a
lower threshold.
Linear filtering would cause more blurring.



THRESHOLD
CONSTANT FRACTION

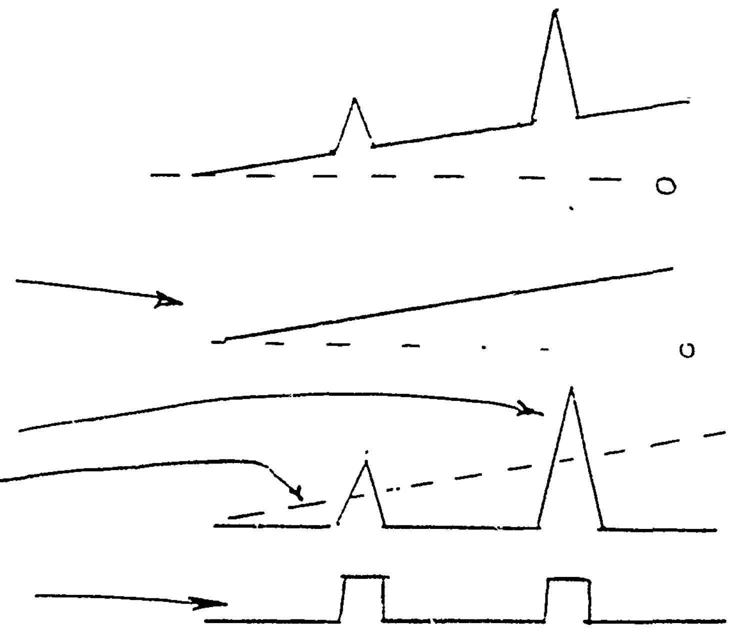
Assume a 3:1 ratio of reflectivity of objects to background.

Image

Background after low-pass filter.

Normalized image
and
background.

Thresholded image

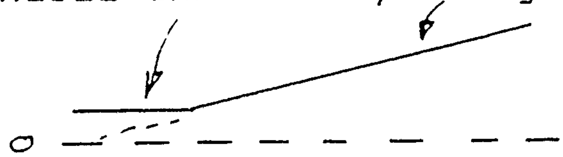


The threshold has the same relative position for different illumination levels.

For other reflectivities, first multiply the background by a constant.

To prevent noise from being thresholded, replace background by:

MAX(noise threshold, background)



Noise does not obey the laws of reflectivity!

THRESHOLD
ADAPTIVE CONSTANT FRACTION

What if reflectivity difference
is small
is not known
changes over the image.

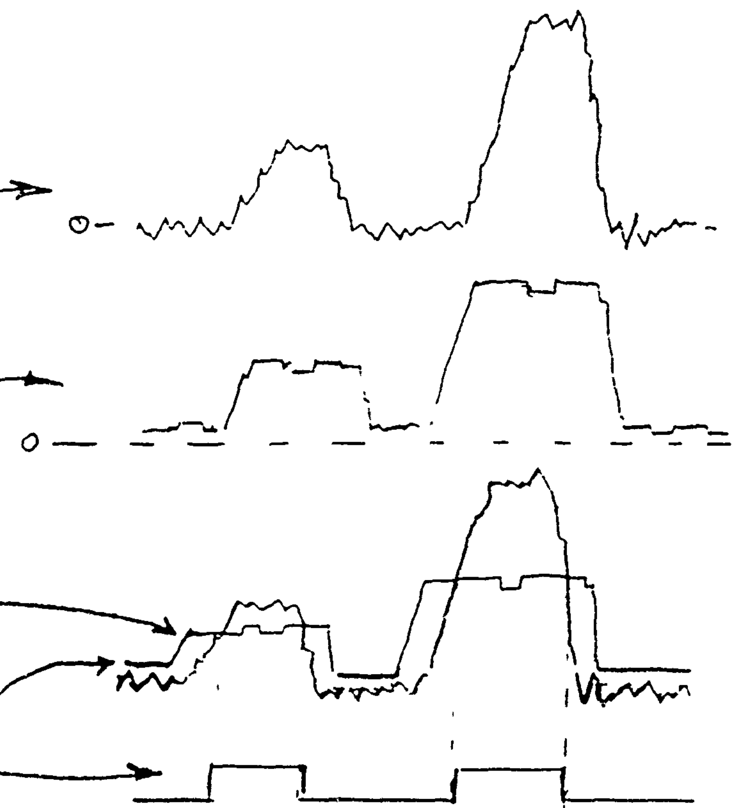


Image after
background
normalization

Dilate
image

Divide dilated
image by 2.

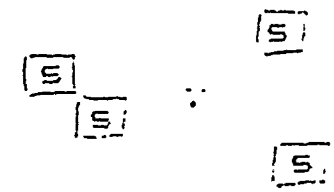
Compare with
original image

To prevent thresholding noise, use:
MAX(noise threshold, (dilated image)/2)

BIT PLANE VECTOR CORRELATION

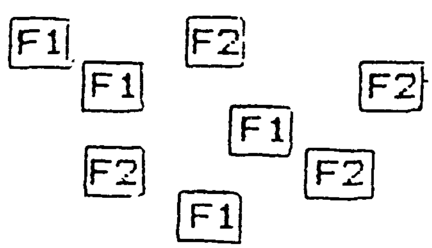
Vector morphology. A type of erosion.

In the usual erosion a structuring element is defined and the AND operation is applied to the image bitplane;

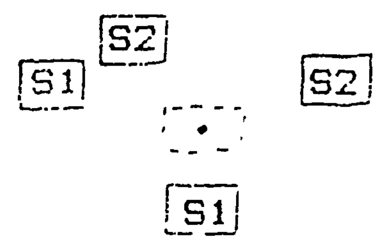


or, very loosely, $I \ominus S = \underset{S}{\mathrm{AND}}\ I$

In vector morphology, the image is several bit planes of features, say two features F1 and F2. The structuring element also has multiple states.



Image



Structuring element

$$I \ominus S = (\underset{S1}{\mathrm{AND}}\ F1)\ \mathrm{AND}\ (\underset{S2}{\mathrm{AND}}\ F2)$$

An example of character recognition is given in a later section.

IMAGE PROCESSING

STATE - OF - THE - ART

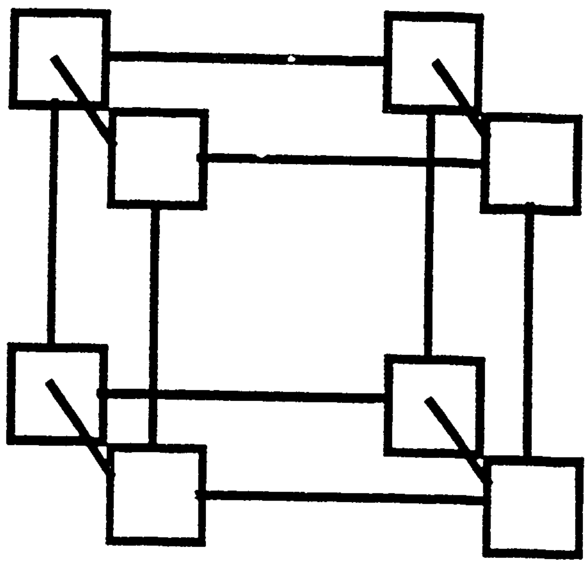
TECHNOLOGY

MUST USE PARALLEL PROCESSORS

MIMD - COARSE GRAINED
SIMD - FINE GRAINED

MIMD

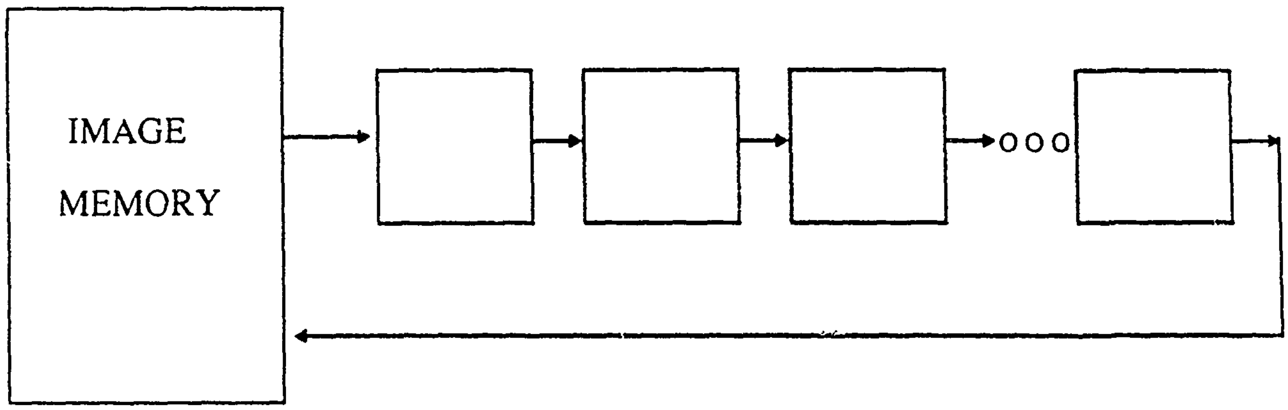
HYPERCUBE



COMPLEX MICROPROCESSOR
SYSTEMS

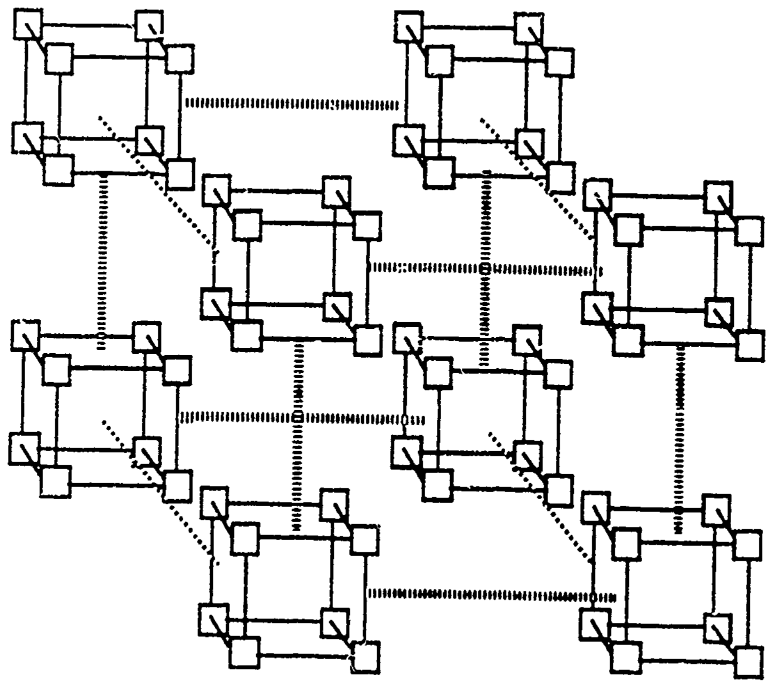
MIMD

PIPELINE



SIMD

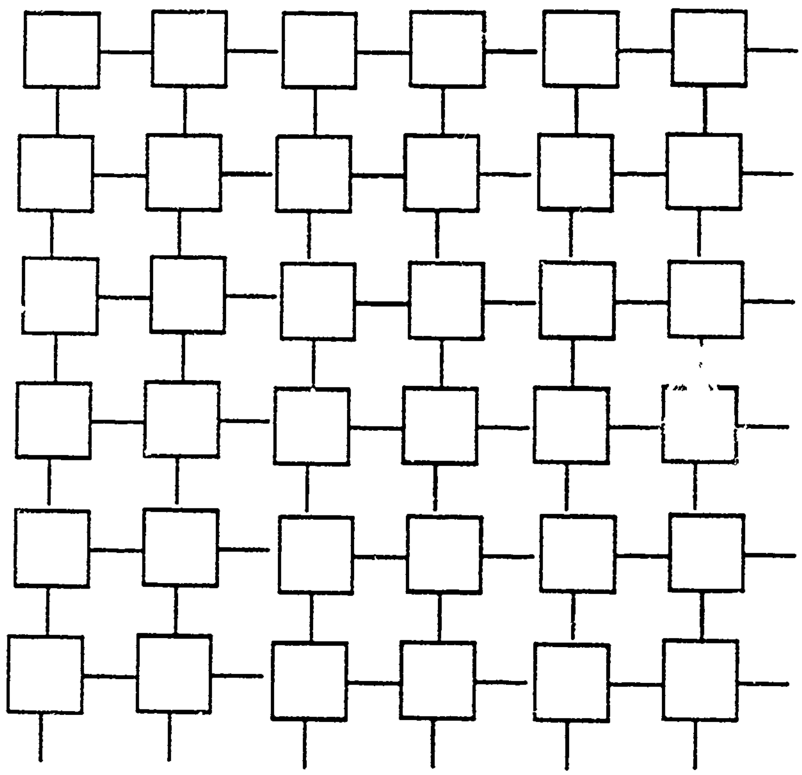
HYPERCUBE



FINE GRAINED PROCESSORS

SIMD

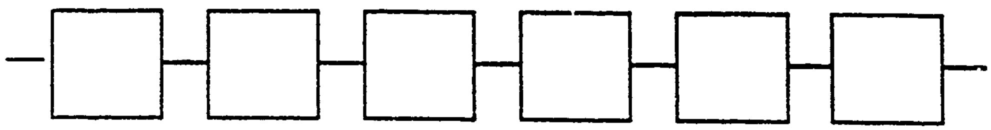
MESH CONNECTED



FINE GRAINED PROCESSORS

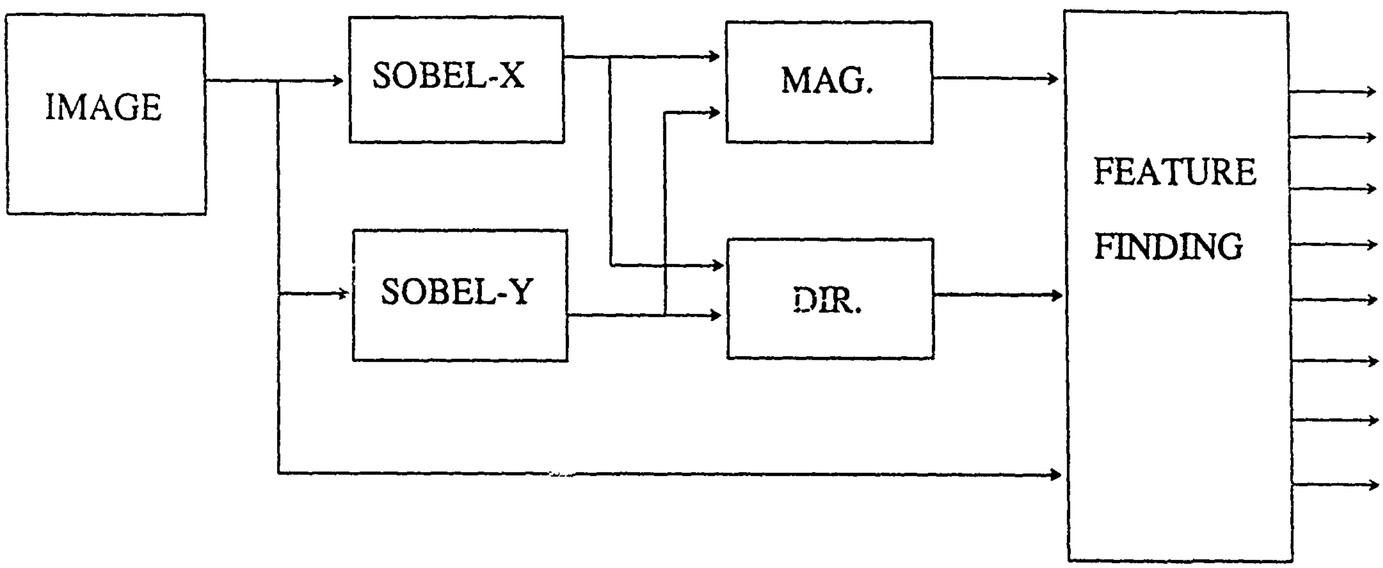
SIMD

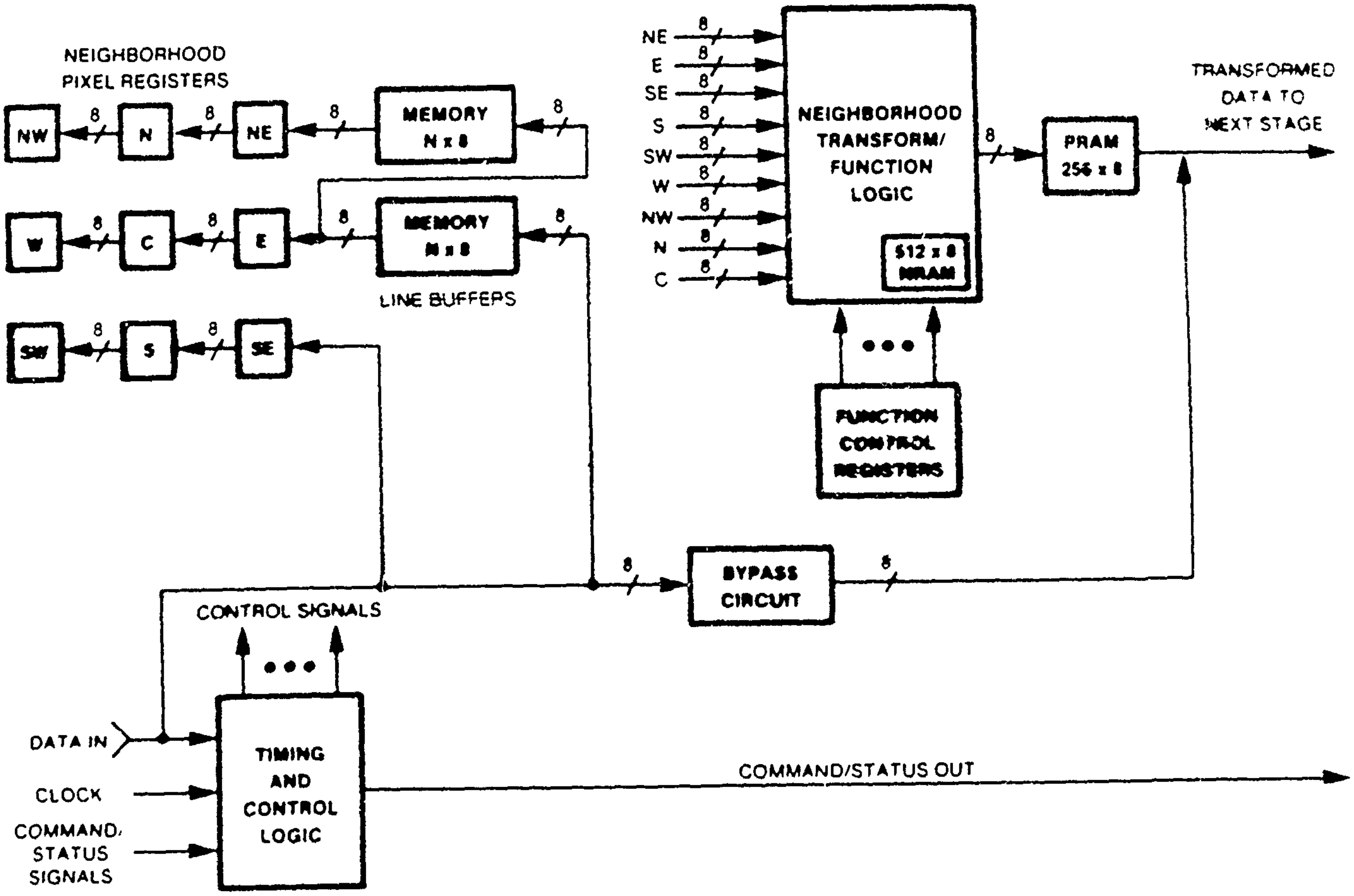
LINEAR ARRAY



AIS - 5000

TYPICAL PROGRAM FLOW

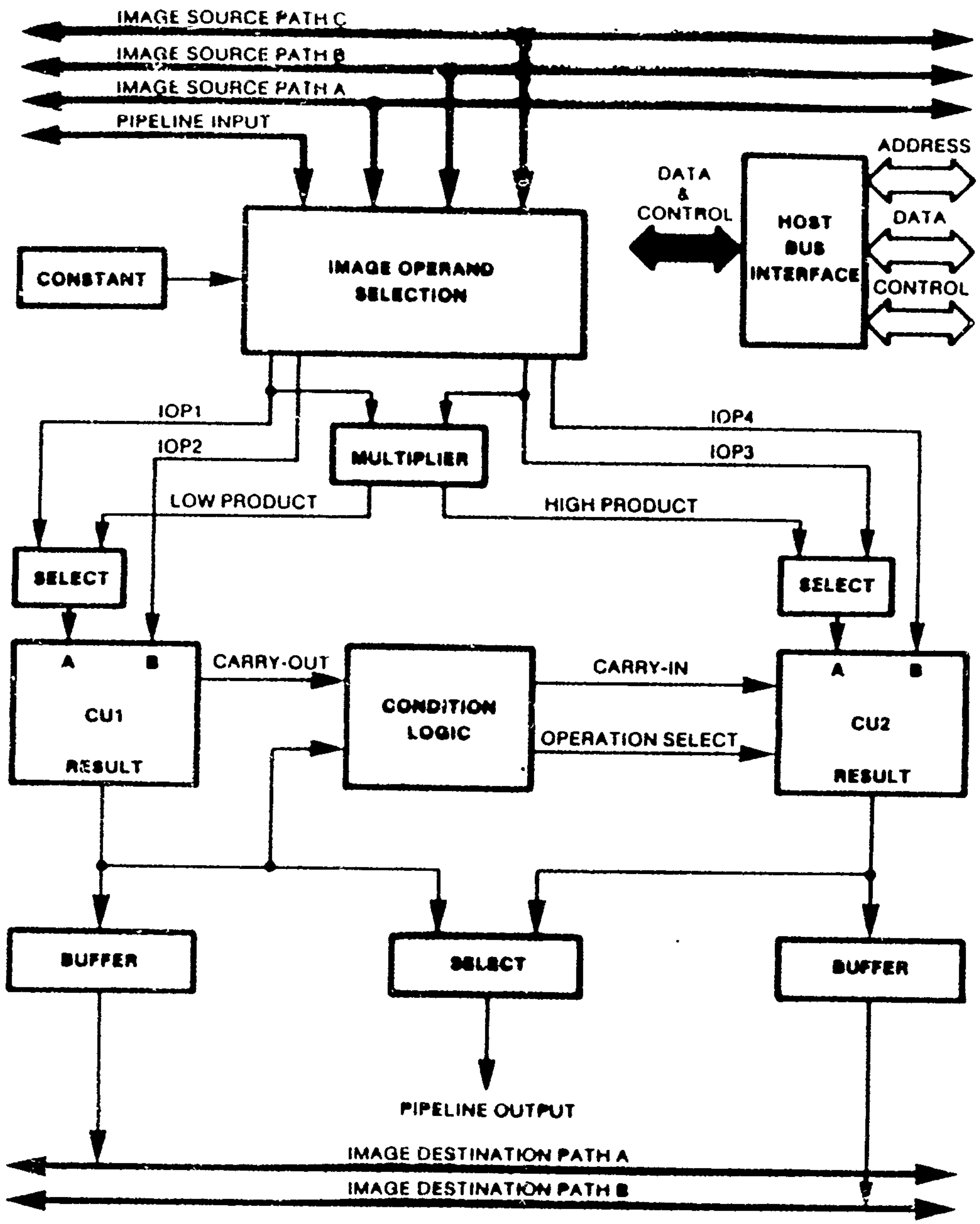




Neighborhood Processing Stage Block Diagram

ENVIRONMENTAL RESEARCH INSTITUTE OF MICHIGAN
P.O. BOX 8618 ANN ARBOR, MI 48107-8618 (313)994-1200 TELEX 4940991 ERIM

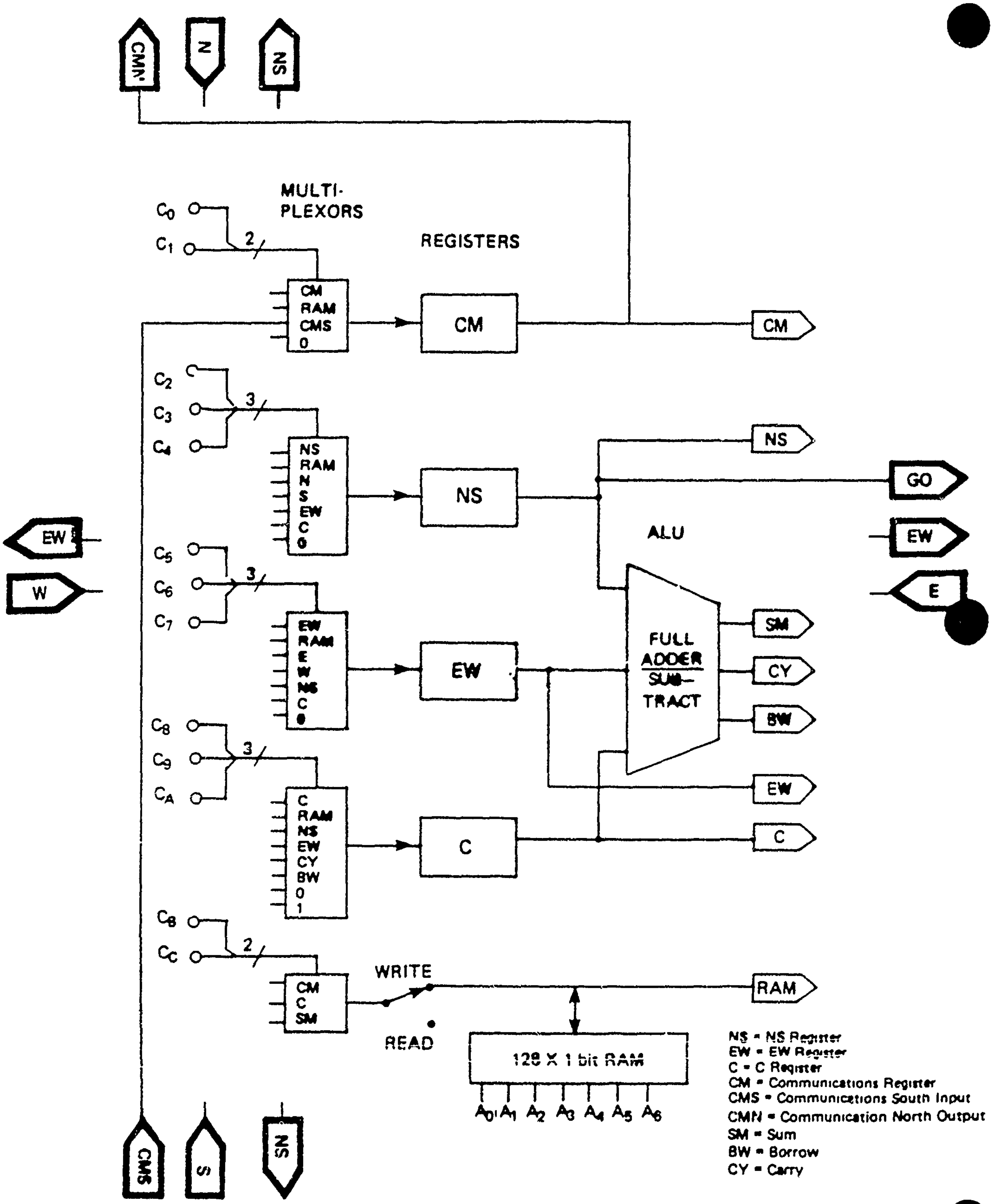
Pipeline



Combiner Block Diagram

ERIM   Pipeline

SCHEMATIC DIAGRAM OF ONE PROCESSOR ELEMENT

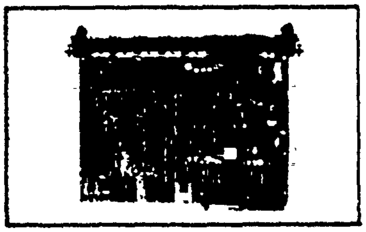


NCR GAPP CHIP

MESH CONNECTED

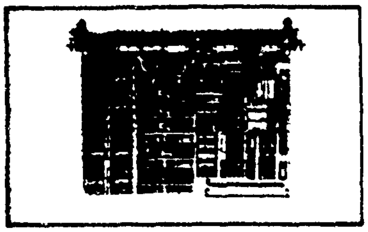
THE
MaxVideo
FAMILY

DIGIMAX



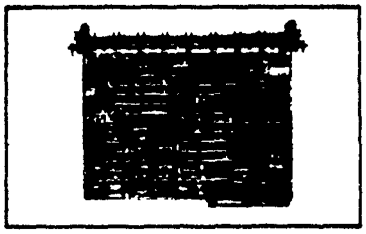
Performs A/D and D/A conversion on RS-170 (60Hz) and CCIR (50Hz) standard video signals in real time. Eight camera inputs are software selectable. 32 banks of input/output Look-Up Tables. Three output D/A channels, and graphics overlay hardware are provided.

VFIR



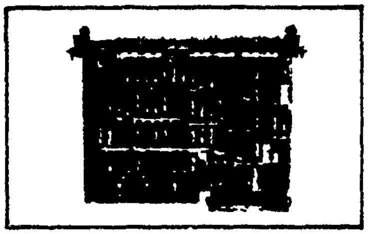
Linear pixel processor performs 3 x 3 two dimensional convolution or 10 x 1 FIR filter on 512 x 512 image in one frame time. 100 million, 20-bit precision multiply accumulates per second. Full 20-bit precision at end of adder tree.

FRAMESTORE



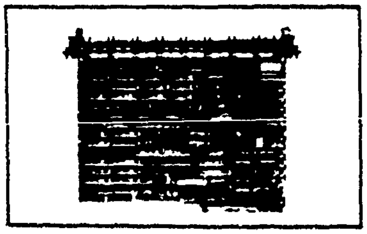
Three complete 8-bit 512 x 512 stores on a single board. Can be utilized as three independent 8-bit buffers or as one 16- or 24-bit deep framestore. Extensive multiplexing for user flexibility.

SNAP



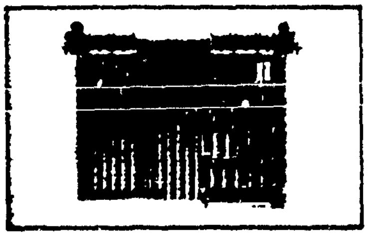
Real time non-linear Systolic Neighborhood Area Processor. Performs 180 million 8-bit comparisons (10 million neighborhoods) per second; mathematical morphology: erosion and dilation algorithm implementation.

MAX-SP



Signal Processor performs full frame single point FIR filtering in one frame time. Multi-image merging, differencing, multiplier, minimum/maximum operations, ALU, clipper unit, barrel shifter and LUTs.

PROTOMAX

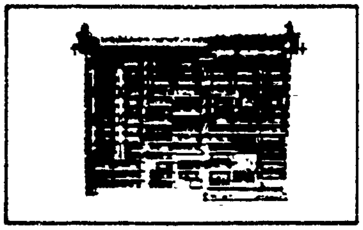


Wire wrap prototyping module for developing MaxVideo compatible designs. PROTOMAX includes interface circuitry and connectors to both MAXbus and P1 connector of VMEbus.

THE MaxVideo FAMILY

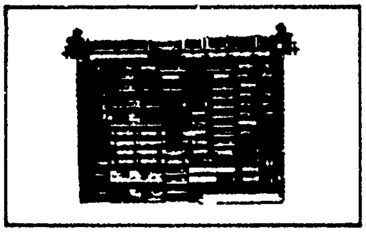
Pipe line

MAX-XFS



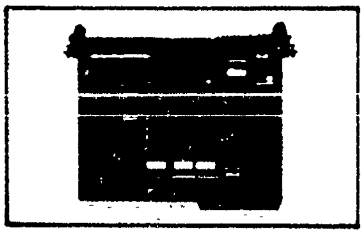
Transposing framestore which contains two complete frames of video storage. Each frame can be read and/or written in row or column order to achieve real time 90 degree image transposition. It is useful for realizing separated horizontal and vertical pipelines.

INTERPOLATOR



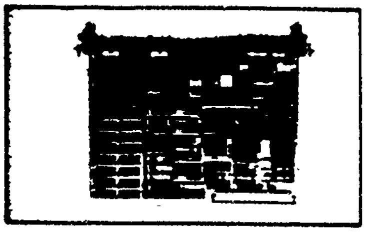
Performs sub-pixel, multirate sampling in real time. It can perform first order transformation in one dimension. Its 8-point aperture and sinc interpolation algorithm yield an extremely precise 16-bit result in conjunction with ADDGEN-1.

ADDGEN-1



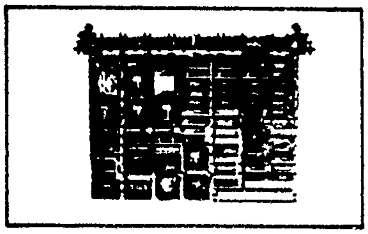
Address generator module which works in conjunction with the INTERPOLATOR module. It creates the addressing necessary to allow INTERPOLATOR to perform first order transformations to 32-bits of spatial resolution. Sub-pixel multirate sampling module.

MAX-GRAPH



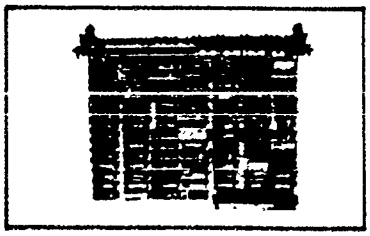
Stand-alone VME graphics controller supporting simultaneous display of 256 colors. Has unique capability of overlaying graphics with real time digital video signals. Implements primitive operations for a variety of geometric draw and fill commands.

MAX-SIGMA



Variable aperture (up to 64 by 256) moving average convolution module. Performs hipass, low pass and band pass filters.

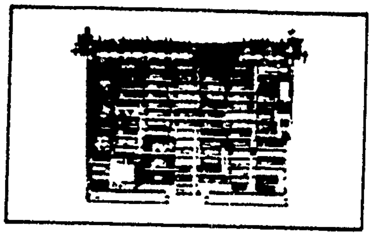
FEATUREMAX



Performs histogram and feature list extraction in real time. Featur. st extraction stores the x, y coordinates of up to 16K spatial grey level specific events.

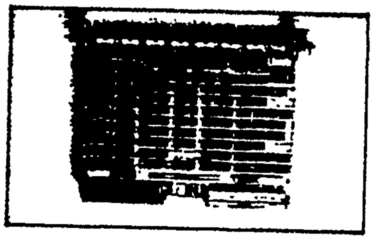
THE
MaxVideo
FAMILY

EUCLID



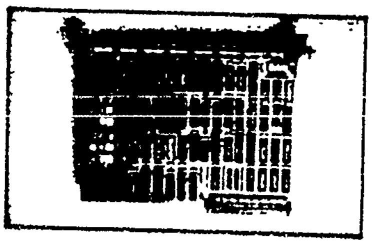
High speed general DSP module. Supported by an extensive preprogrammed "C" callable library and a complete complement of development tools including an ANSI standard "C" compiler. Concurrent data movement and processing and MAXbus compatibility lead a long list of standard features.

ROI-STORE



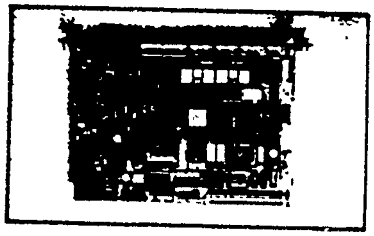
A true advancement on data/imaging storage. Ranging in capacities from 512K bytes to 2 megabytes and operating on user defined regions of interest. Hardware supported pan, scroll and zoom features allow for greater programming flexibility.

MAX-MUX



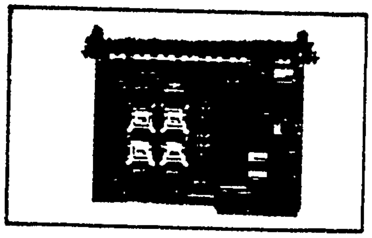
MAX-MUX is a digital cross point switch for the MAXbus. Under user control MAX-MUX allows for the assembly of a more flexible MaxVideo based processing system. A 16 x 16 LUT performs any arbitrary point transformation.

MAX-SCAN



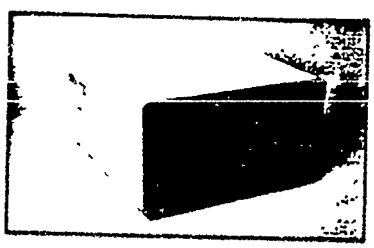
Analog and digital acquisition module. Completely programmable acquisition rates from DC to 20 MHz. Extensive and flexible synchronous options allow interface to numerous devices.

VFIR-MK II



Second generation Video Finite Impulse Response Filter. Implements a 64-point arbitrary coefficient convolution/correlation on a 10 MHz stream. 640 million multiply-accumulates per second allows for real-time 8 x 8 or 64 x 1 operations.

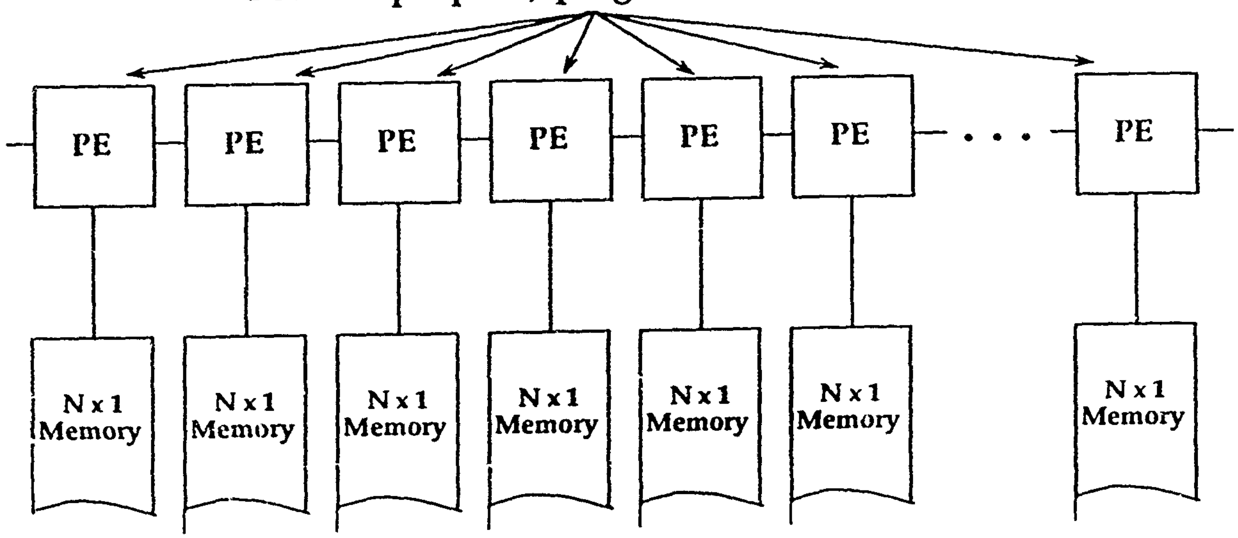
MAX–BOX



Max-BOX is a twenty slot VME chassis featuring 750 watt power supply, forced air cooling, high efficiency plenum design and standard dual height Eurocard compatibility. Designed to house high performance VME modules in an environment suitable for ruggedized use.

System Technology
Linear SIMD Array

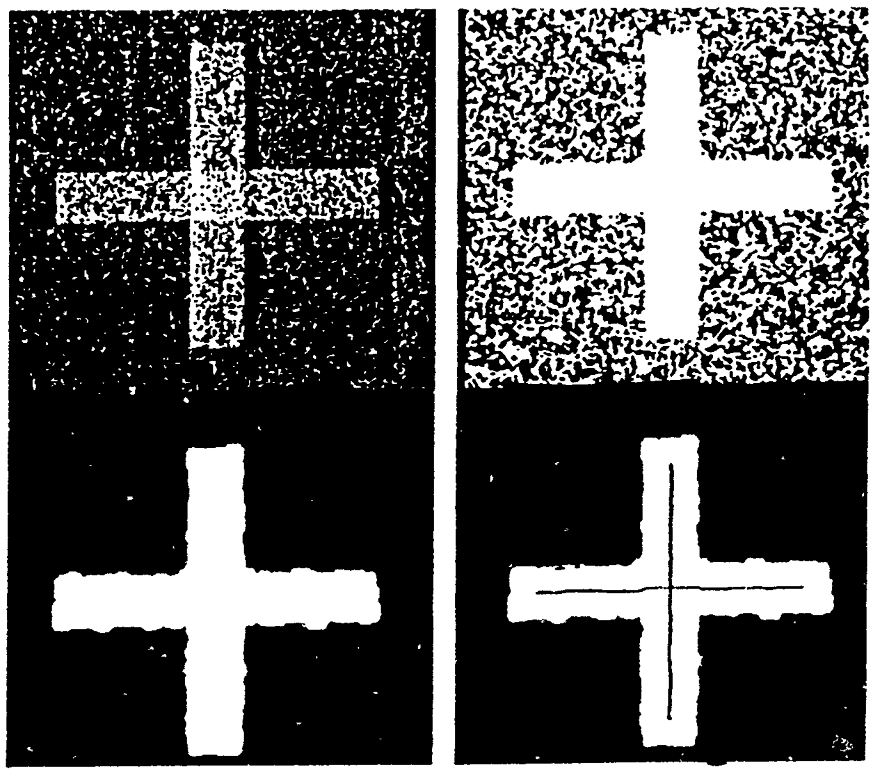
General purpose, programmable, bit serial CPUs



Fine-grained Massive Parallelism

Applied Intelligent Systems, Inc.

Performance Benchmark
Abingdon Cross



Applied Intelligent Systems, Inc.

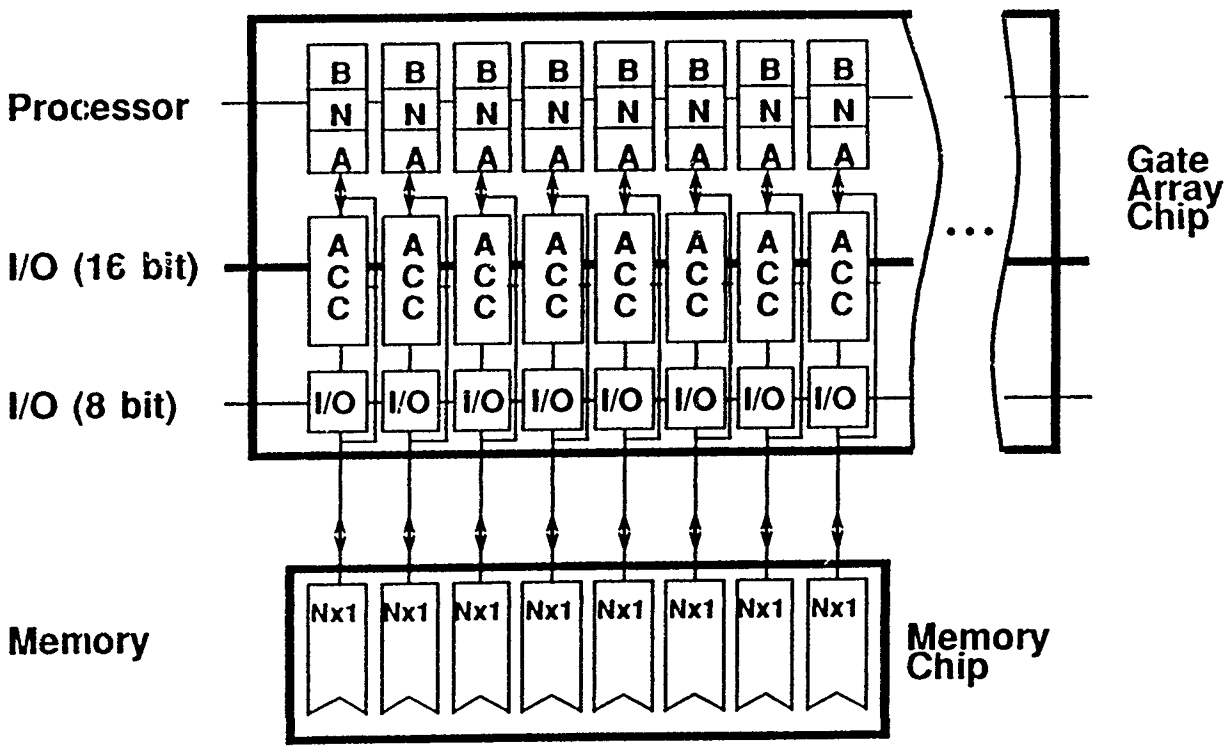
Architecture Comparison
Other Massively Parallel Machines

The AISI machines can be compared with other fine-grained, massively parallel SIMD array machines

- **Individual PE's are very powerful**
  - Fully programmable and general purpose PE's are more powerful than GAPP, MPP or Connection machine
- **Controller can be simpler than for 2D parallel array**
  - Don't have to produce array instructions at nanosecond rate
- **Virtual 2D array is efficient and easy to support**
  - Don't have to partition data sets - simple conceptual model of processing
  - No overhead for North/South communication

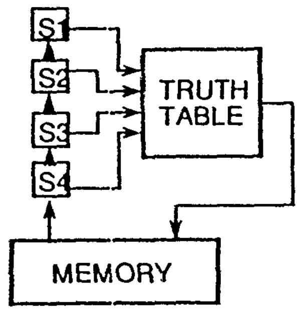
Applied Intelligent Systems, Inc.

Chip Technology
Centipede

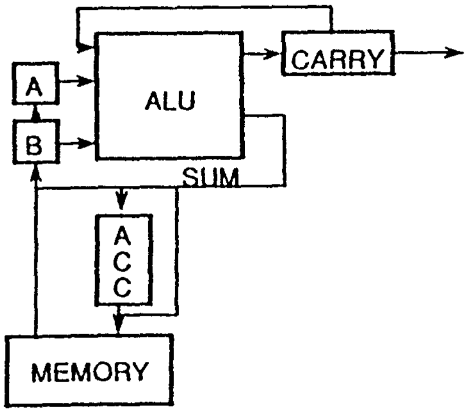


Applied Intelligent Systems, Inc.

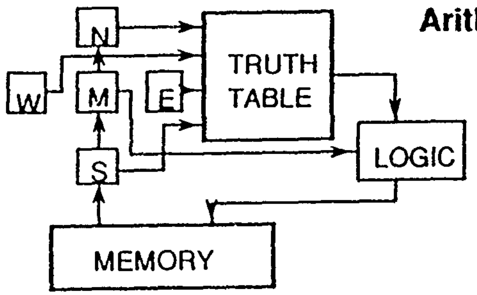
Centipede Processor Operations



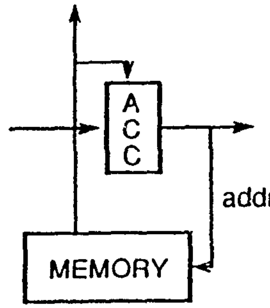
Logical Operations



Arithmetic Operations



Neighborhood Operations



Indirect Addressing

Applied Intelligent Systems, Inc.

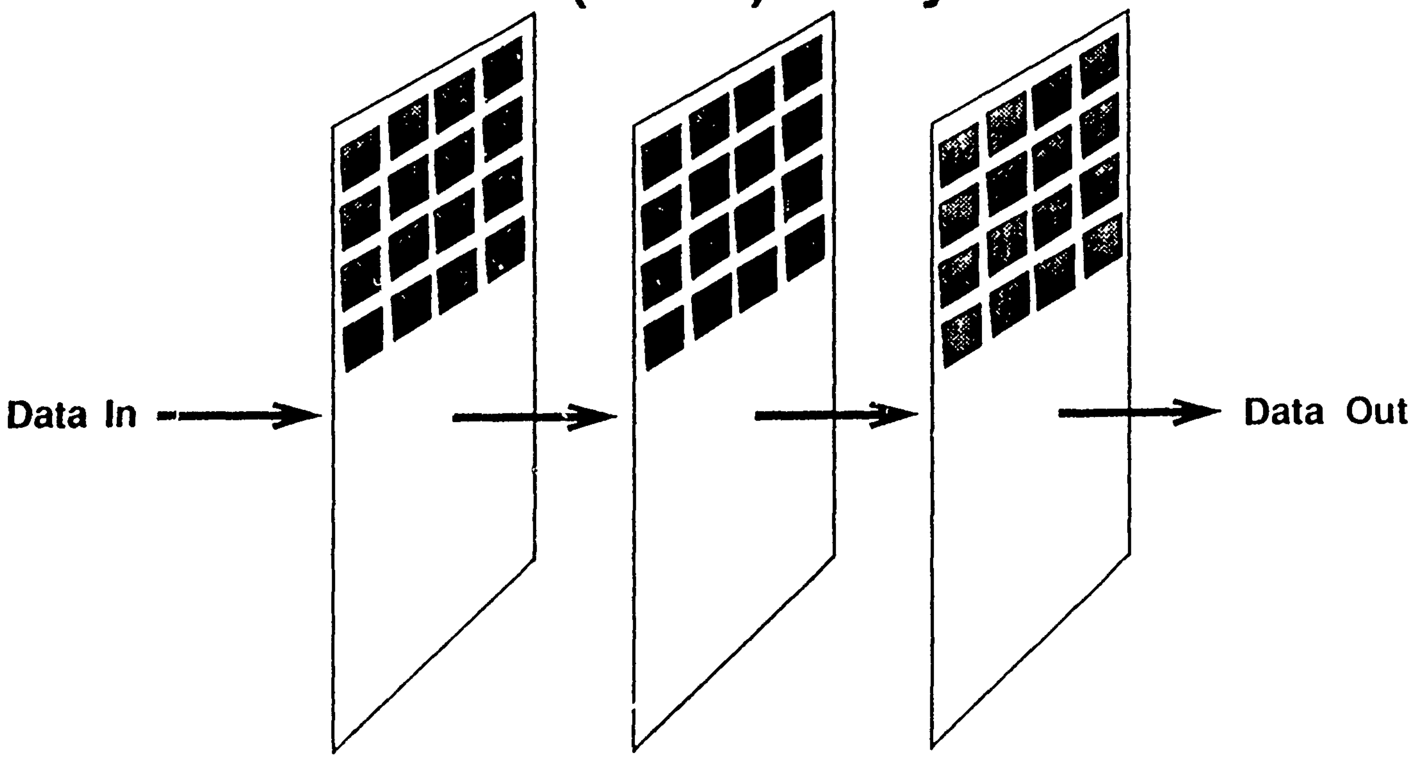
Centipede Performance

512 x 512 Images (512 PEs), 20 MHz Clock

	Time (msec)
■ Binary Erosion or Dilation (48 pixel radius disk)	0.6
■ Add 2 Images	0.65
■ Feature Extraction (whole image)	0.85
■ Histogram (8 bit full image)	3.3
■ Convolve with 3 x 3 Kernel (unsymetric)	7.9
■ Sliding Disk - 48 pixel radius	9.0
■ Rolling ball - 48 pixel radius	95.0
■ 2D FFT (complex 16 bit)	500.0

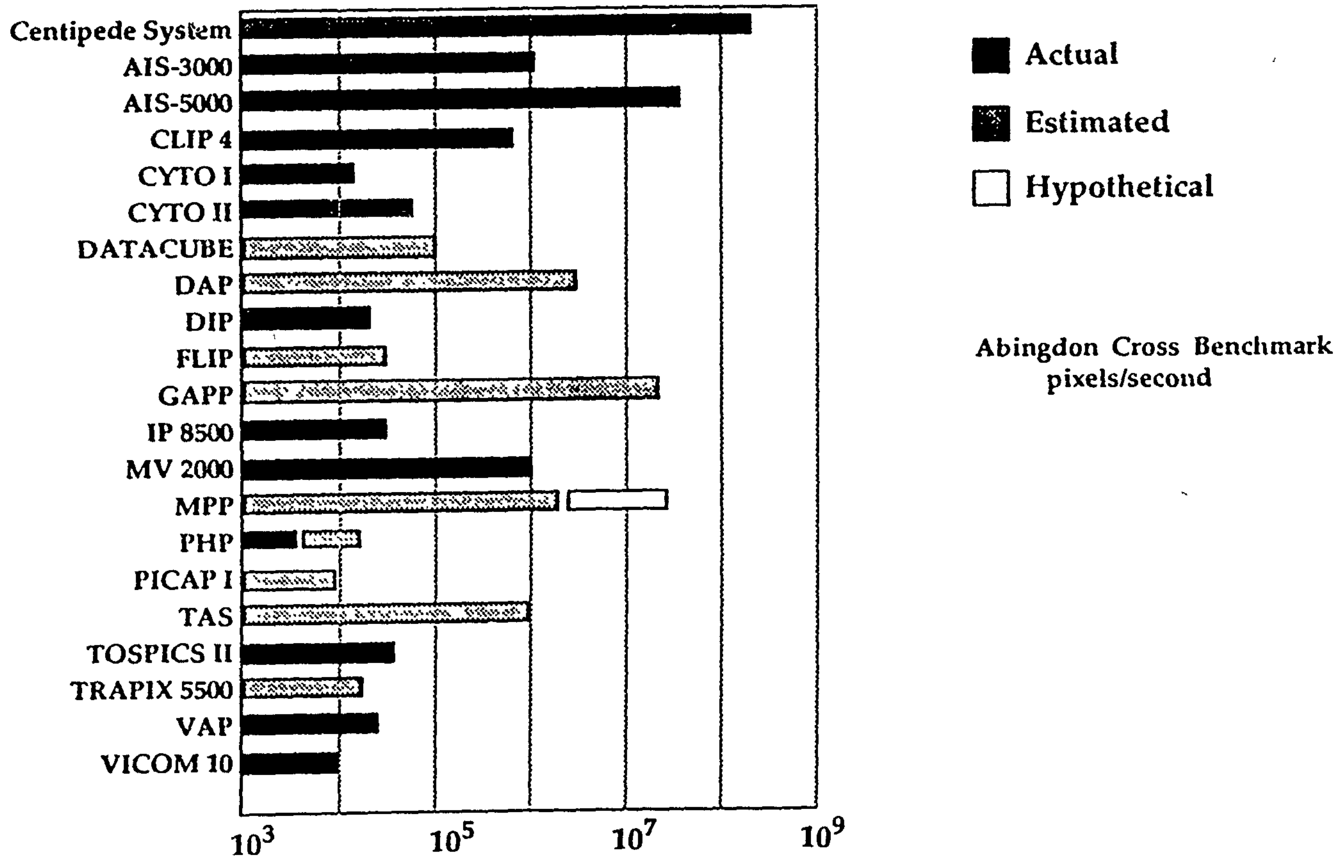
Applied Intelligent Systems, Inc.

Centipede System M(SIMD) Array



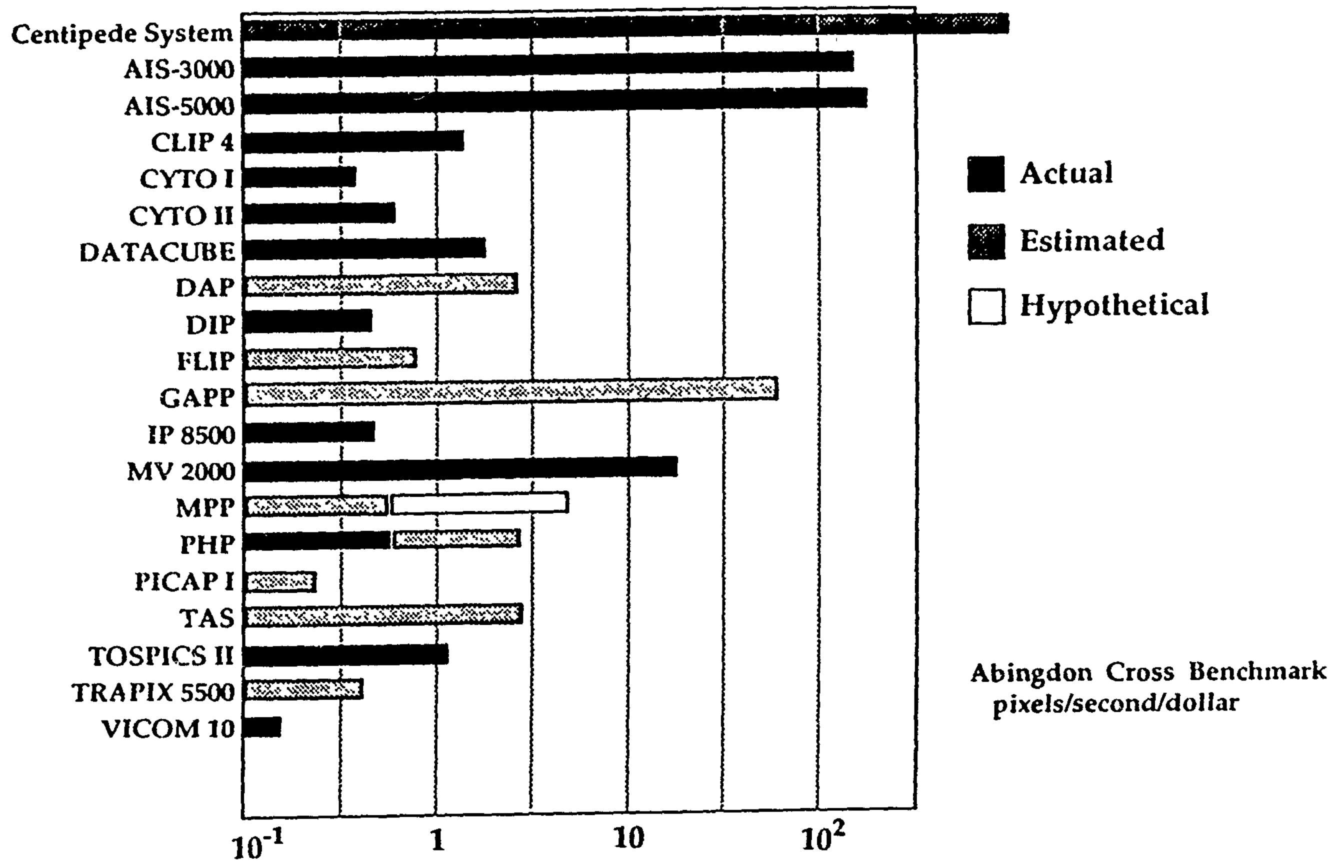
Applied Intelligent Systems, Inc.

Performance Advantage



Applied Intelligent Systems, Inc.

Price/Performance Advantage



Applied Intelligent Systems, Inc.

Summary of Manipulation requirements

- "Natural" Representation of signals
- Abstract data objects and inquiry operations
- Control structure for sequencing through rule base
- Representation of symbolic information (i.e. extensivity, idempotency, ...) about systems.

Part of ARO 26131.1-EL-CF

MATHEMATICAL MORPHOLOGY APPLIED TO MULTI-SCALE IMAGE REPRESENTATION AND SHAPE DESCRIPTION*

by

Petros Maragos

Division of Applied Sciences

Harvard University

Cambridge, MA 02138

*Talk given at the *Workshop on Mathematical Morphology*, Tom Bevill Center, Huntsville, Alabama, July 25-26, 1988.

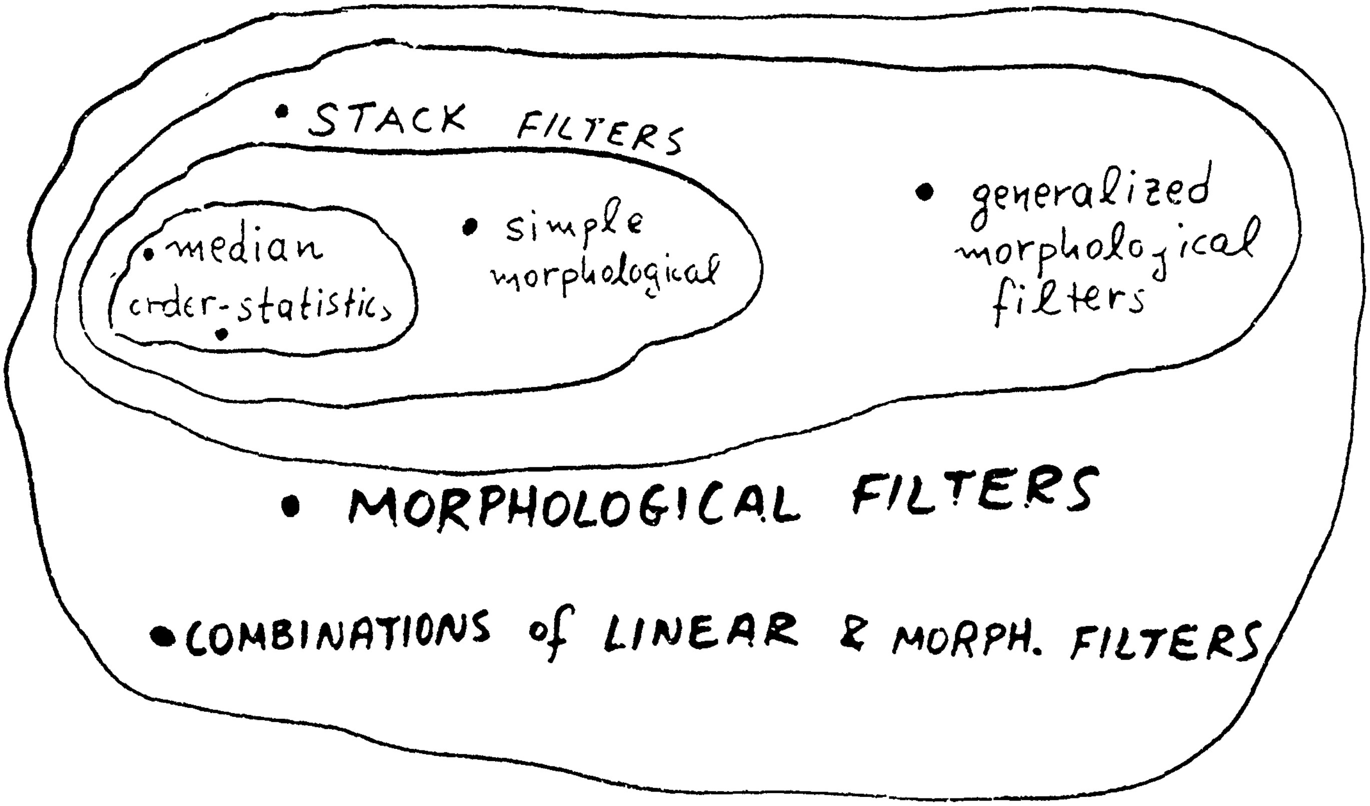
MATHEMATICAL MORPHOLOGY APPLIED TO MULTI-SCALE IMAGE REPRESENTATION AND SHAPE DESCRIPTION

Petros Maragos

Division of Applied Sciences, Harvard University

- Multi-scale nonlinear image smoothing
- Pattern Spectrum
- Skeleton Representation & Coding of Images (with R. Schafer)
- Symbolic Image Modeling
- Fractals & morphology (with R. Libeskind)

NONLINEAR FILTERS w/ SIGNAL-SHAPING PROPERTI[illegible]



* STATISTICAL PROPERTIES
* SPREAD-SPECTRUM EFFECT
* LOGICAL-SYNTACTICAL PROPERTIES

input 0 0 0 6 0 0 1 2 1 0 1 2 1 0 2 4 2 0 2 4 2 0 0 1 2 3 4 4 4 4 4 0 0 [illegible]

3-average 0 0 2 2 $2\frac{1}{3}$ 1 $\frac{4}{3}$ 1 $\frac{2}{3}$ 1 $\frac{4}{3}$ 1 1 2 $\frac{8}{3}$ [illegible] 2 $\frac{8}{3}$ 2 [illegible] $\frac{1}{3}$ 1 2 3 [illegible] 4 4 4 $\frac{8}{3}$ $\frac{4}{3}$ 0 [illegible]

3-median 0 0 0 0 0 0 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 0 0 1 2 3 4 4 4 4 4 0 0 [illegible]

	LINEAR	MEDIAN
noise spikes	blur	eliminate
oscillation	weaken	flatten
step edges	blur	preserve
ramp edges	blur	preserve

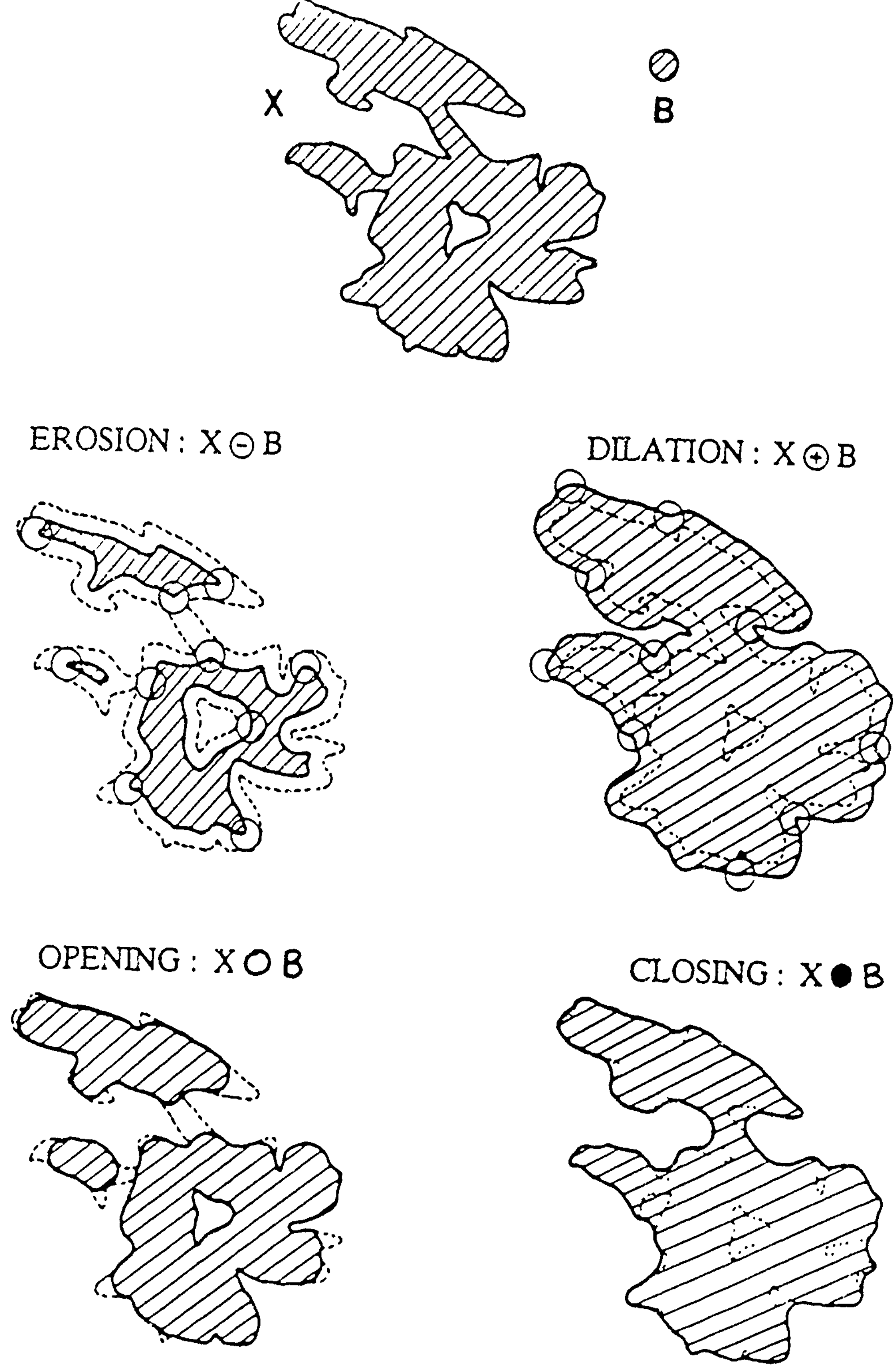


FIGURE 2

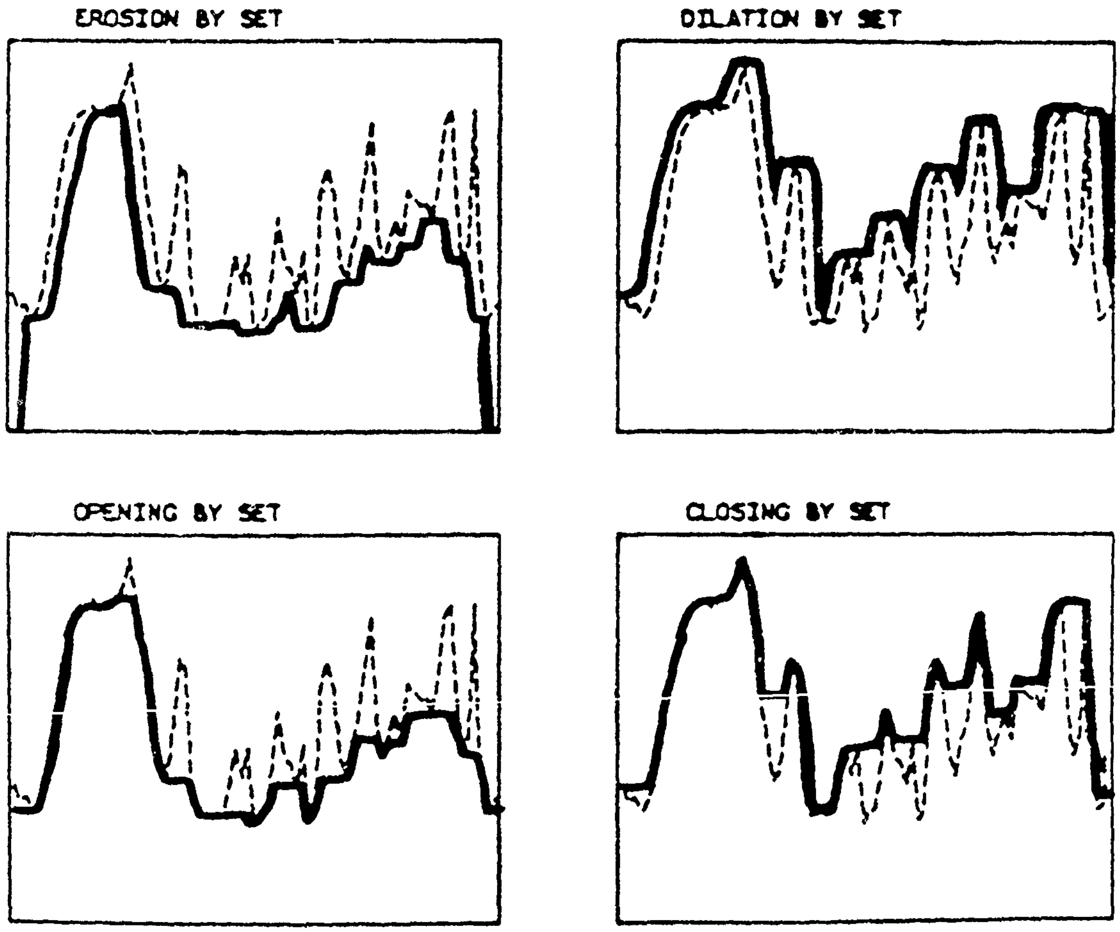
MORPHOLOGICAL FUNCTION TRANSFORMATION

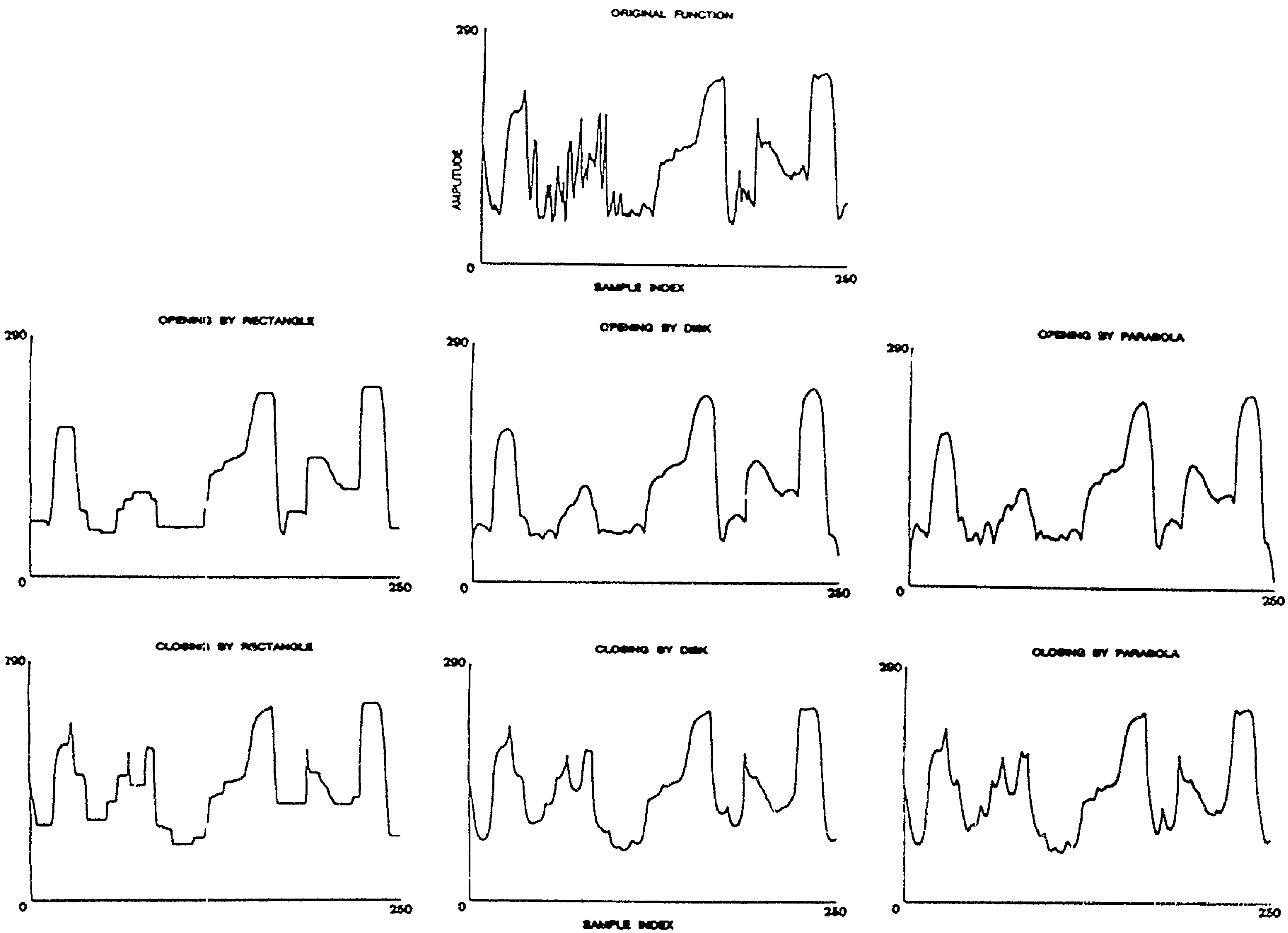
Erosion: $f \ominus g\,(x) = \underset{y}{\mathrm{MIN}} \{ f(x+y) - g(y) \}$

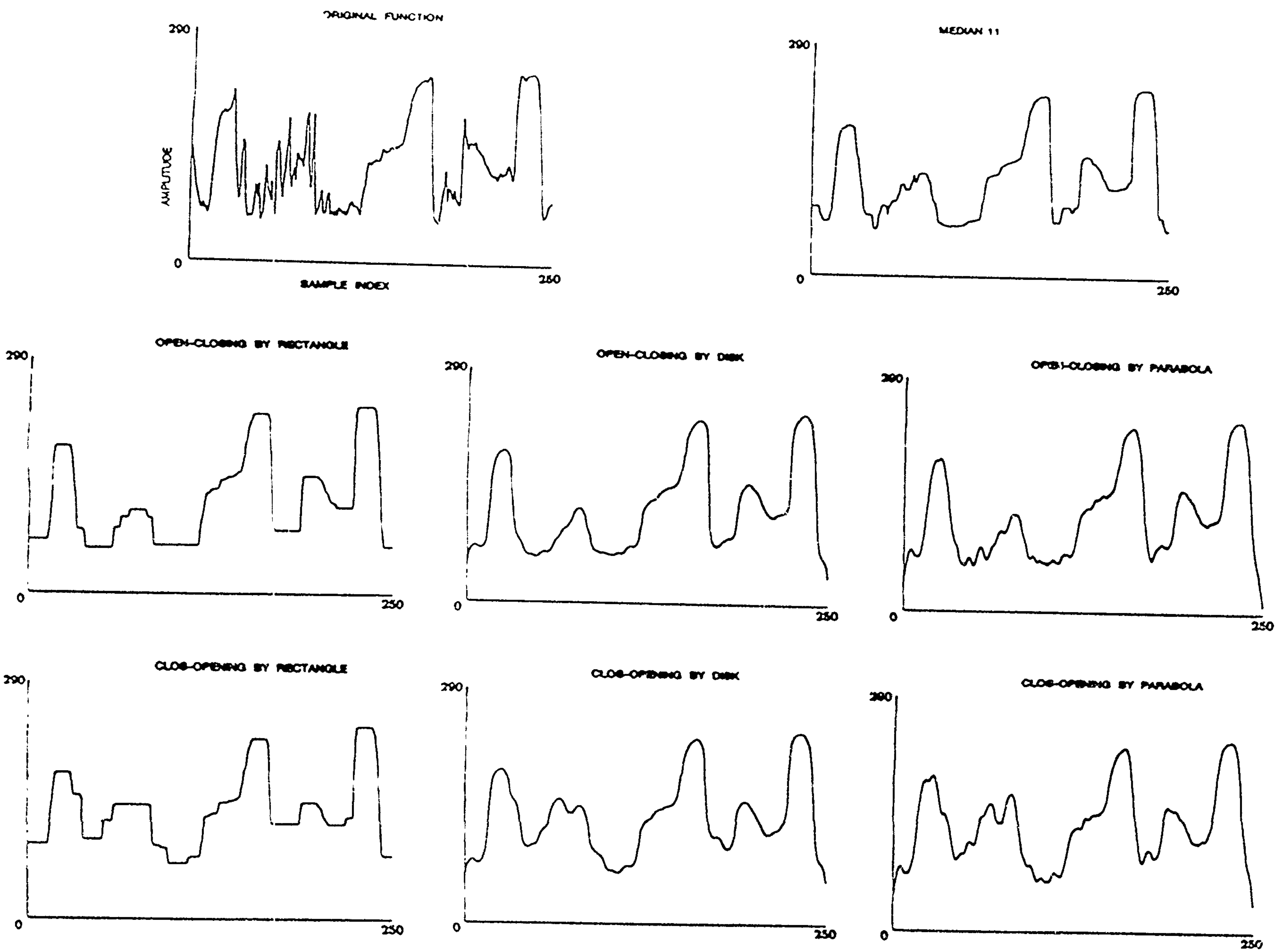
Dilation: $f \oplus g\,(x) = \underset{y}{\mathrm{MAX}} \{ f(x-y) + g(y) \}$

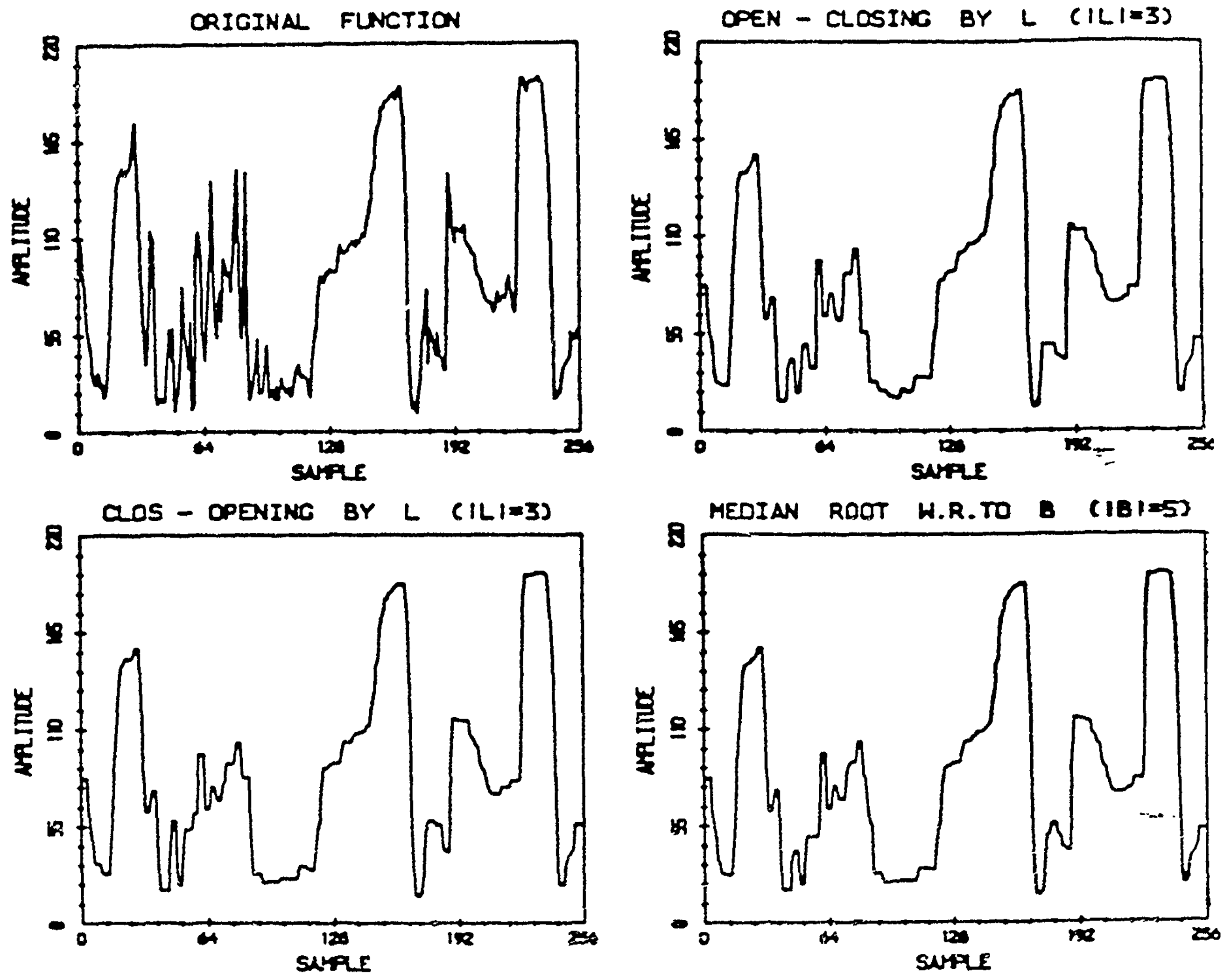
Opening: $f \circ g = (f \ominus g) \oplus g$

Closing: $f \bullet g = (f \oplus g) \ominus g$




ORIGINAL FUNCTION
290
AMPLITUDE
0
SAMPLE INDEX
250
OPENING BY RECTANGLE
OPENING BY DISK
OPENING BY PARABOLA
CLOSING BY RECTANGLE
CLOSING BY DISK
CLOSING BY PARABOLA


ORIGINAL FUNCTION
290
AMPLITUDE
0
SAMPLE INDEX
250
MEDIAN 11
290
0
250
OPEN-CLOSING BY RECTANGLE
290
0
250
OPEN-CLOSING BY DISK
290
0
250
OPEN-CLOSING BY PARABOLA
290
0
250
CLOS-OPENING BY RECTANGLE
290
0
250
CLOS-OPENING BY DISK
290
0
250
CLOS-OPENING BY PARABOLA
290
0
250



ORIGINAL FUNCTION
OPEN - CLOSING BY L (|L|=3)
CLOS - OPENING BY L (|L|=3)
MEDIAN ROOT W.R.TO B (|B|=5)
AMPLITUDE
SAMPLE
0
64
128
192
256
55
110
165
220

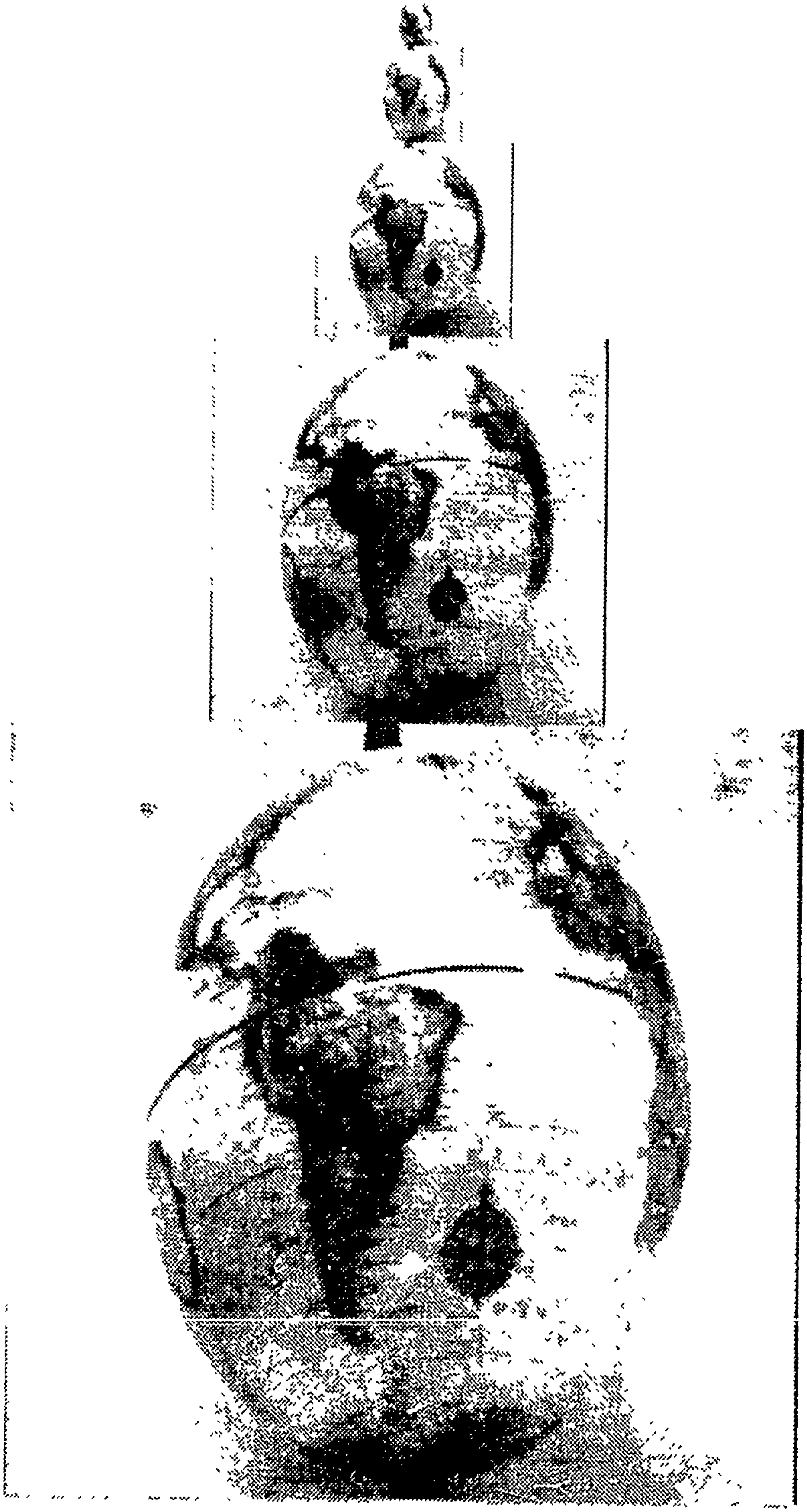
ORIGINAL

smoothing via opening

by [illegible]

GAUSSIAN
SMOOTHING

MORPHOLOGICAL PYRAMID



SMOOTHING FILTERS

- LINEAR:
  - moving average
  - Gaussian
  - Low-pass FIR / IIR

- NONLINEAR
  - α-trimmed mean
  - median
  - opening/closing

noise suppression
edge preservation / no shift

- direct relation to size
- prototypes of size distributions..
- multi-scale smoothing
- pattern spectrum
- multi-scale shape representation
- symbolic image representation.

Mather.

UNIFIED REPRESENTATION THEORY

Theorem: Every transl.-invariant, increasing, (upper-semicontinuous) filter is a supremum of erosions (or infimum of dilations).

Median:

$$\mathrm{med}\{f(x-1), f(x), f(x+1)\} = \mathrm{MAX}\begin{Bmatrix} \min\{f(x-1), f(x)\} \\ \min\{f(x), f(x+1)\} \\ \min\{f(x-1), f(x+1)\} \end{Bmatrix}$$

Opening:

$$[\text{OPEN } f \text{ by } \{-1,0,1\}](x) = \mathrm{MAX}\begin{Bmatrix} \min[f(x-2), f(x-1), f(x)] \\ \min[f(x-1), f(x), f(x+1)] \\ \min[f(x), f(x+1), f(x+2)] \end{Bmatrix}$$

Moving Average:

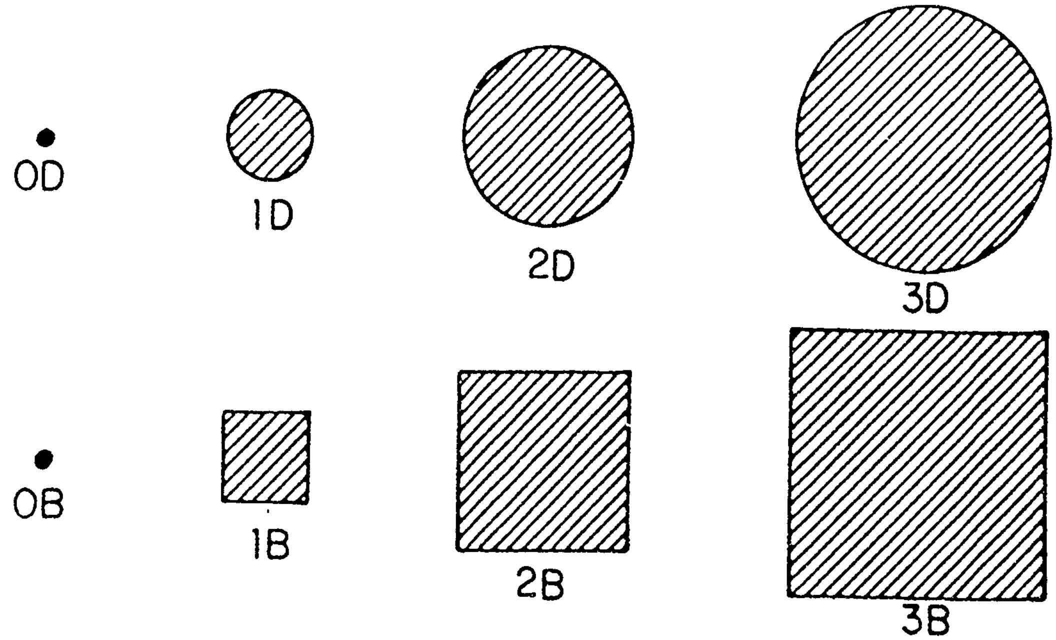
$$\frac{f(x)+f(x-1)}{2} = \sup_{r\in\mathbb{R}}\left\{\min[f(x)+r,\ f(x-1)-r]\right\}$$

Convol. w. Gaussian $G(x)$:

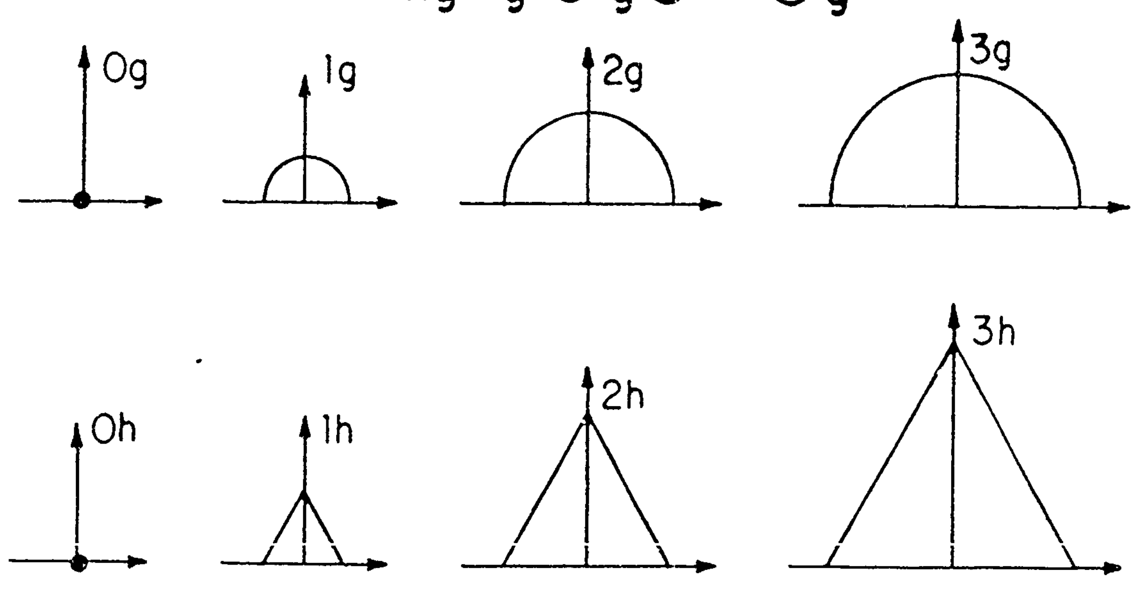
$$f(x) * G(x) = \sup_{\substack{h(\cdot) \\ h*G(0)\geq 0}}\left\{\inf_y [f(y) - h(y-x)]\right\}$$

SIZE

SET DILATION: $nB = \underbrace{B \oplus B \oplus \cdots \oplus B}_{n \text{ times}}$



FUNCTION DILATION: $ng = g \oplus g \oplus \cdots \oplus g$



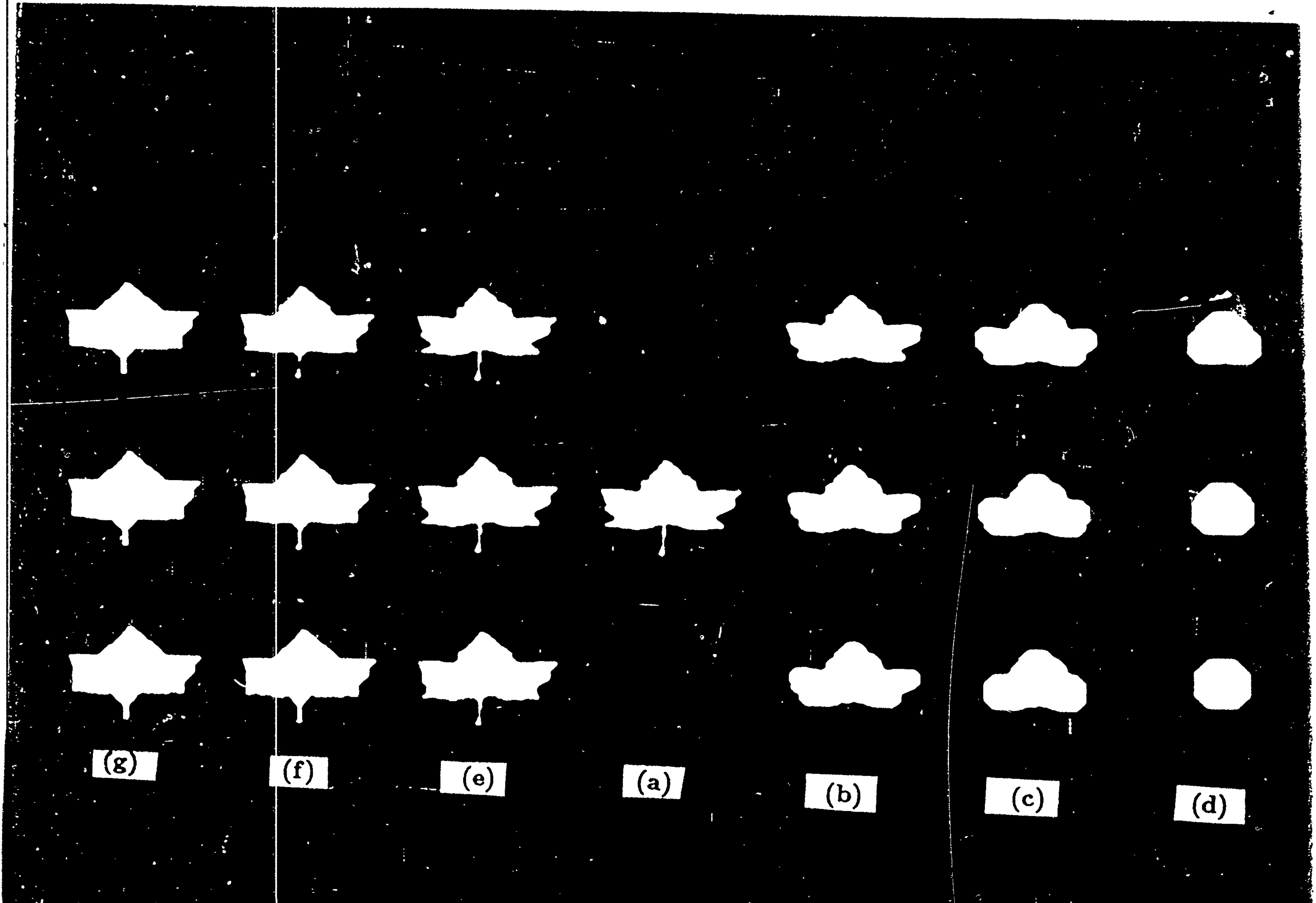


FIGURE 3

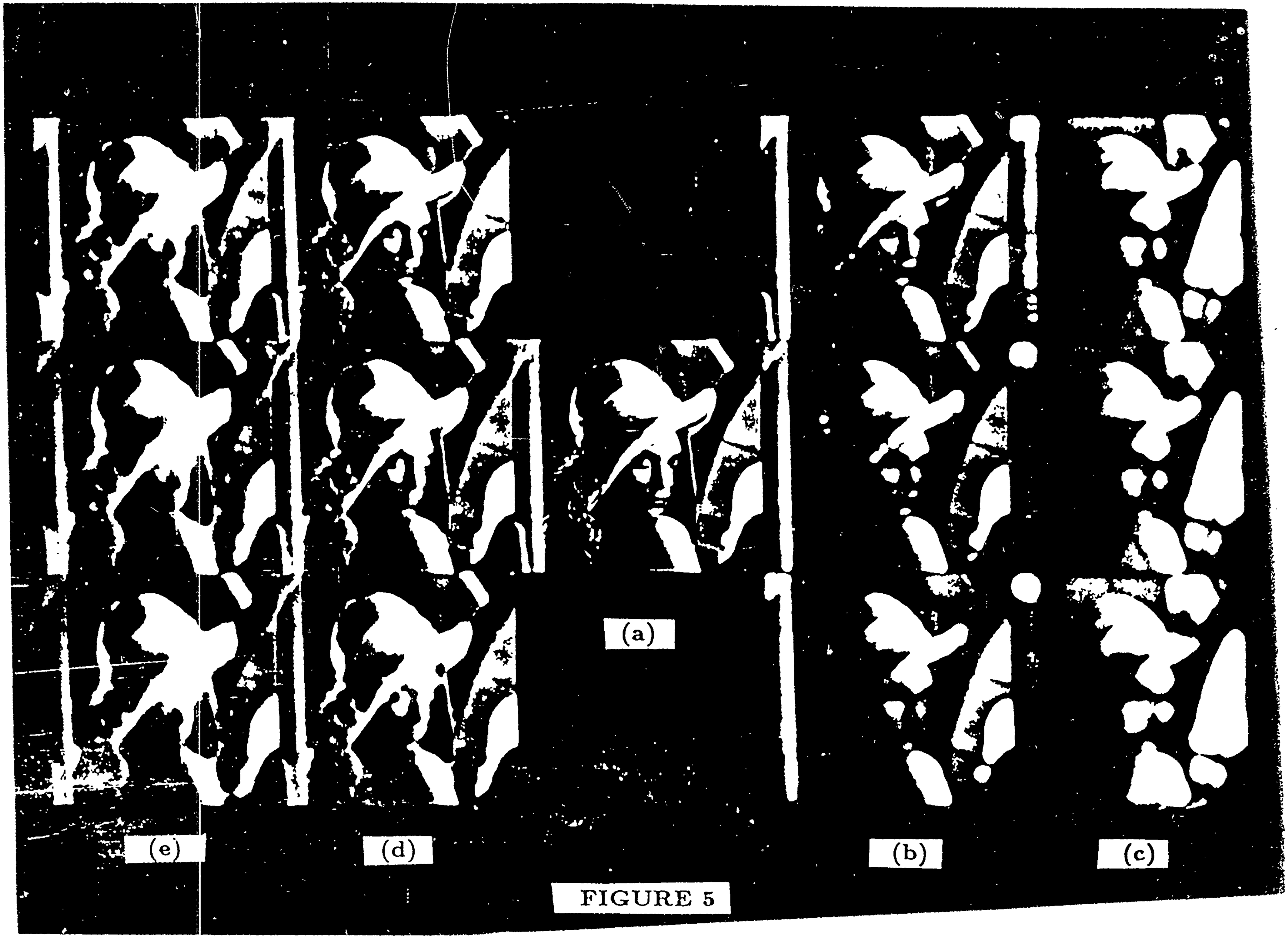


FIGURE 5

The infamous plant!

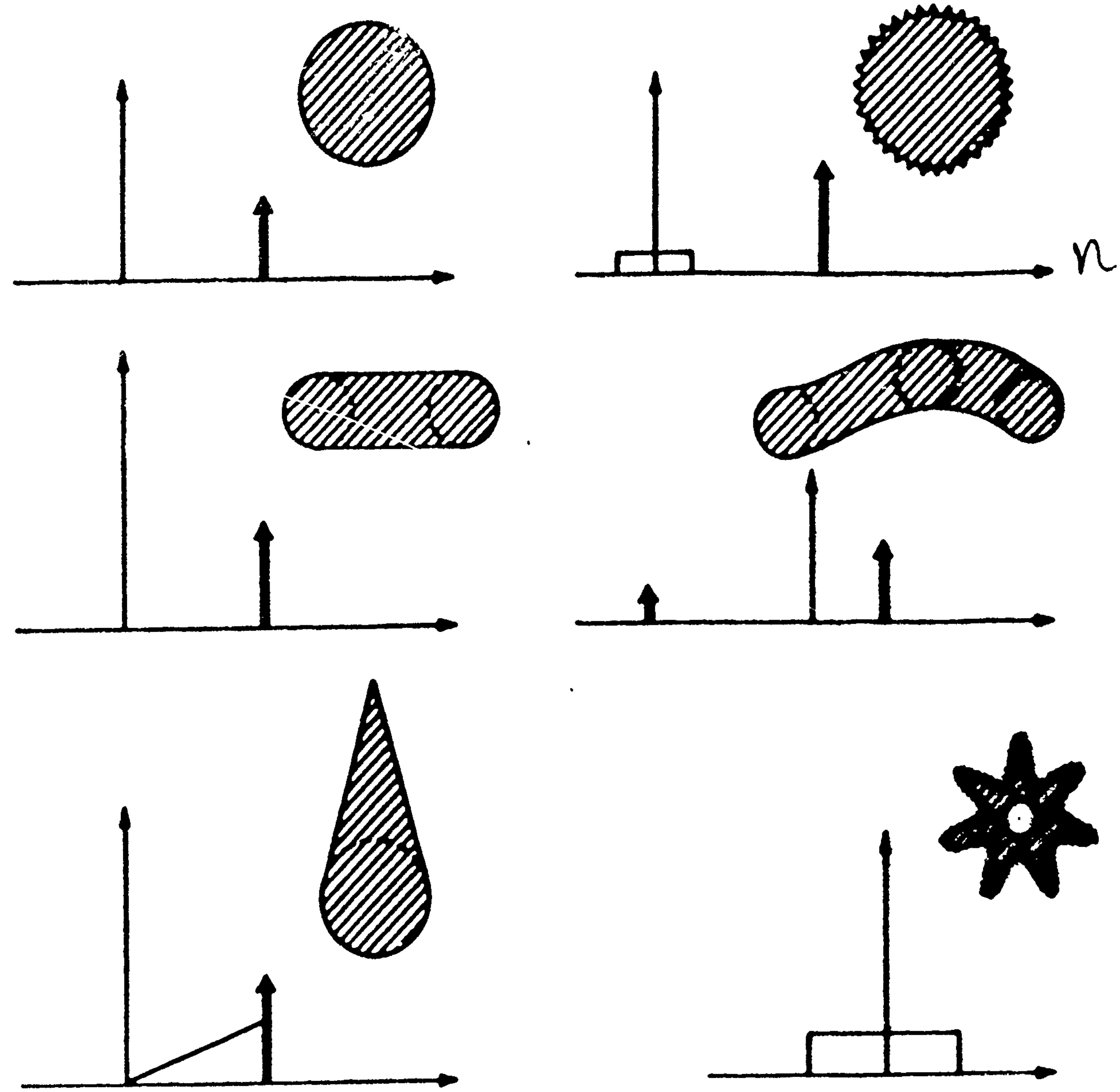
CLOSING BY A 2-D STRUCT ELEMENT (OCTAGON of SIZE .

[illegible] 2

MIN-of-CLOSINGS by 4 1-D LINEAR SEGMENTS of SIZE 8

8

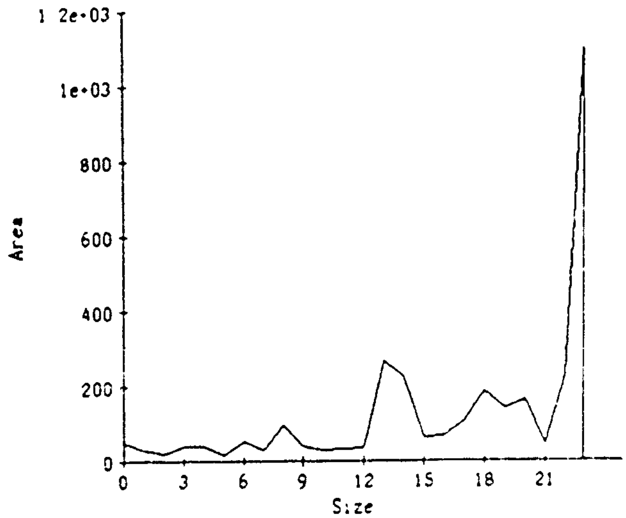
PATTERN SPECTRA



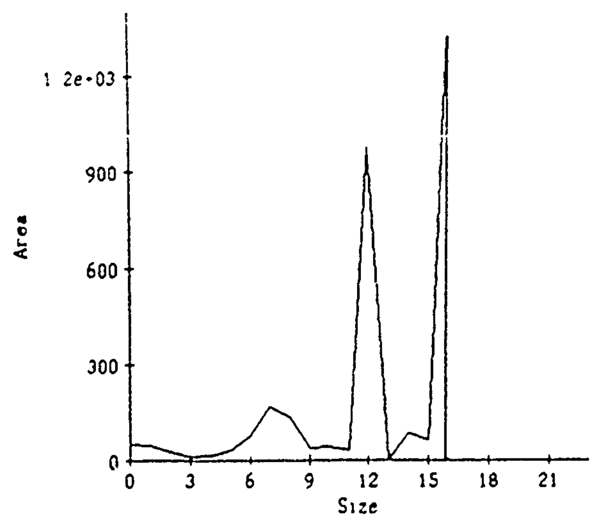
$X \ddot{o} (n+1)B \subseteq X \circ nB \subseteq \cdots \subseteq X$

Pattern Spectrum: $PS(n,B) = A[X \circ nB - X \circ (n+1)B],\ n \geq$

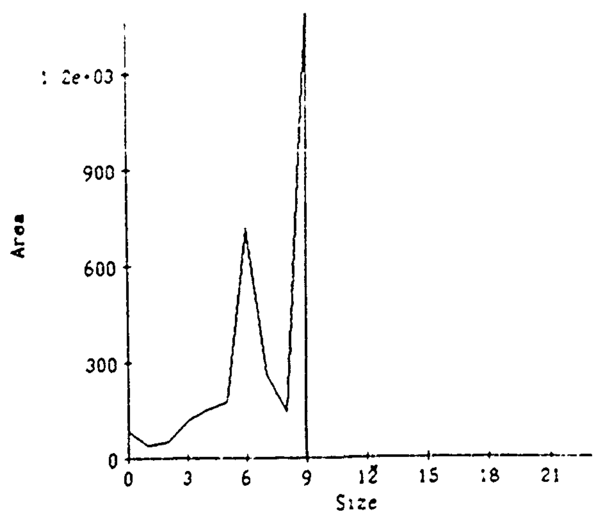
Pattern Spectrum of Leaf w.r.t. rhombus

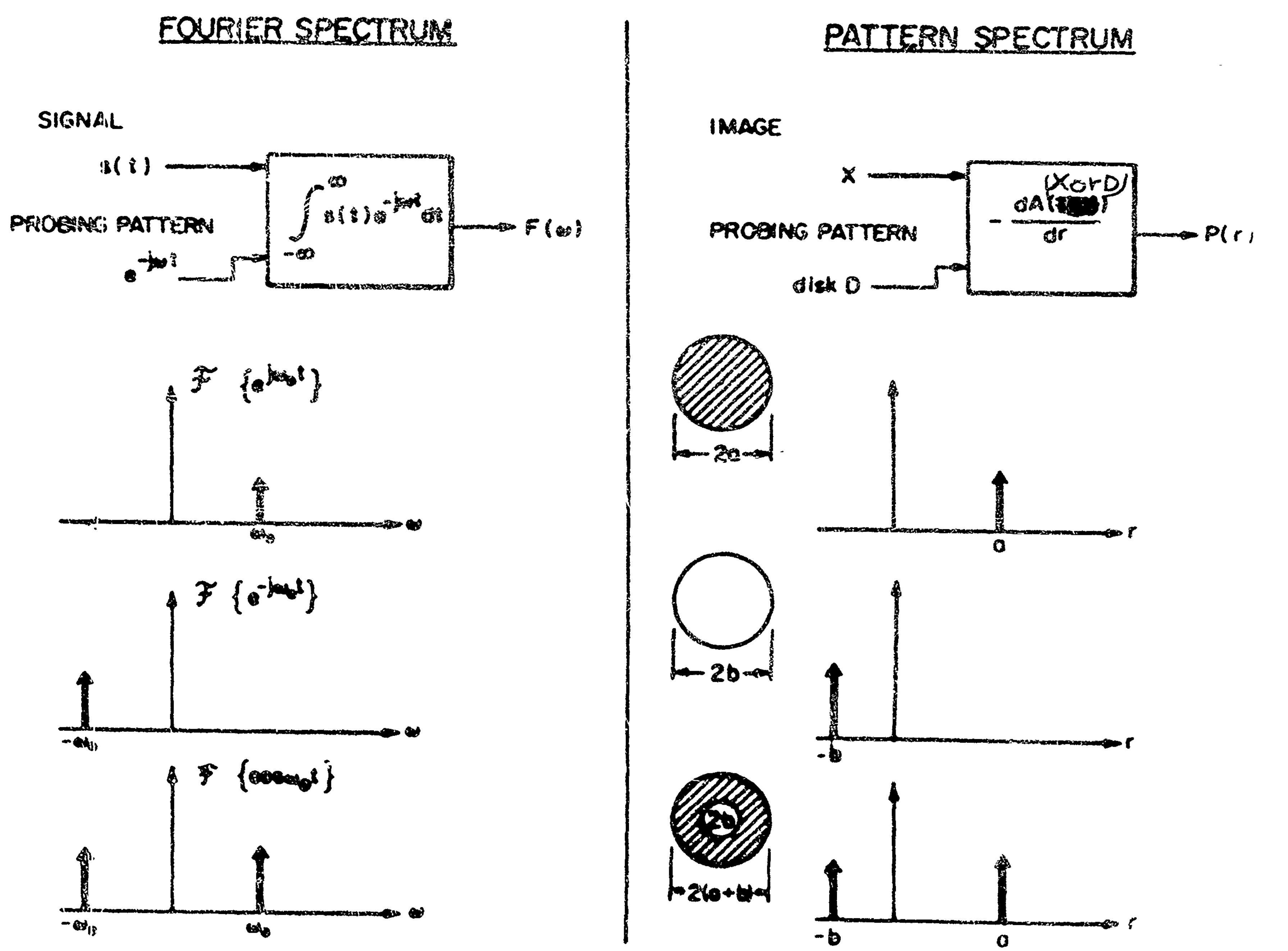


square-9



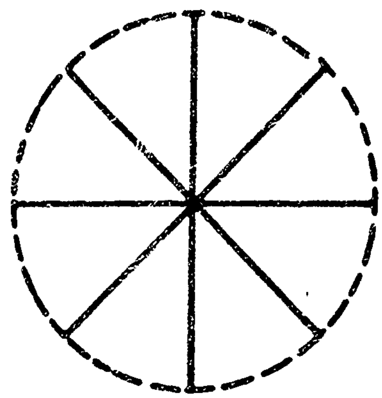
octagon

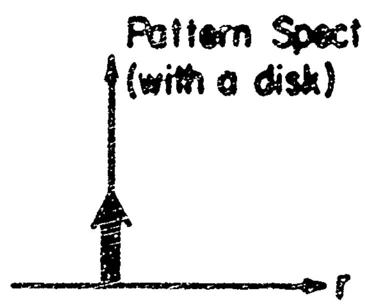


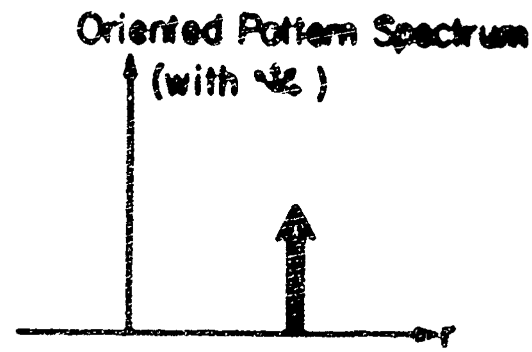


FOURIER SPECTRUM
SIGNAL
s(t)
PROBING PATTERN
$e^{-j\omega t}$
$\int_{-\infty}^{\infty} s(t)e^{-j\omega t}dt$
F(ω)
$\mathcal{F}\{e^{j\omega_0 t}\}$
ω_0
$\mathcal{F}\{e^{-j\omega_0 t}\}$
$-\omega_0$
$\mathcal{F}\{\cos\omega_0 t\}$
$-\omega_0$
ω_0
ω
PATTERN SPECTRUM
IMAGE
X
PROBING PATTERN
disk D
(XorD)
$-\frac{dA}{dr}$
P(r)
2a
2b
2(a+b)
2b
a
-b
r

ORIENTED PATTERN SPECTRUM

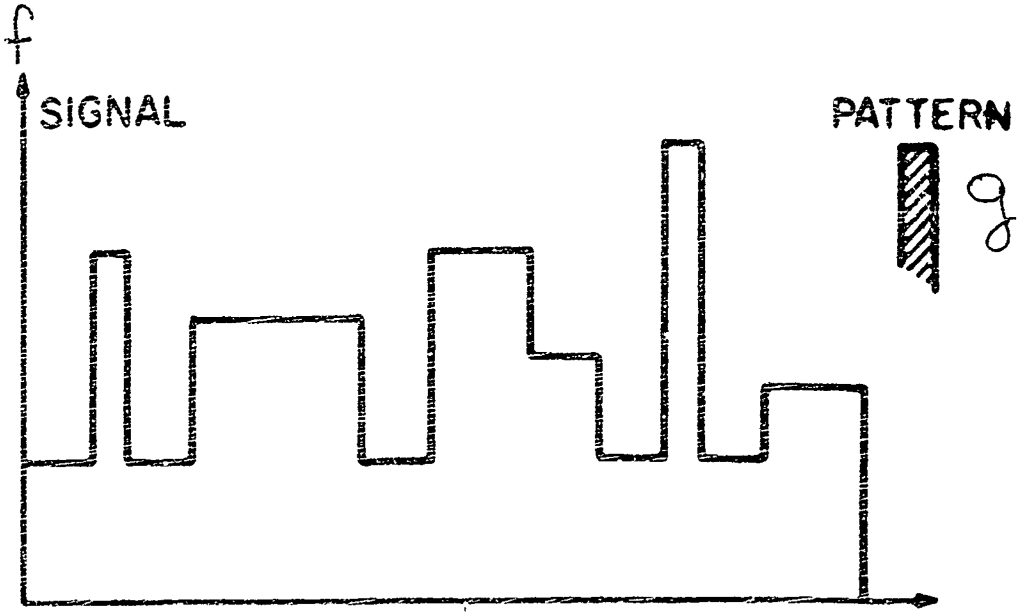


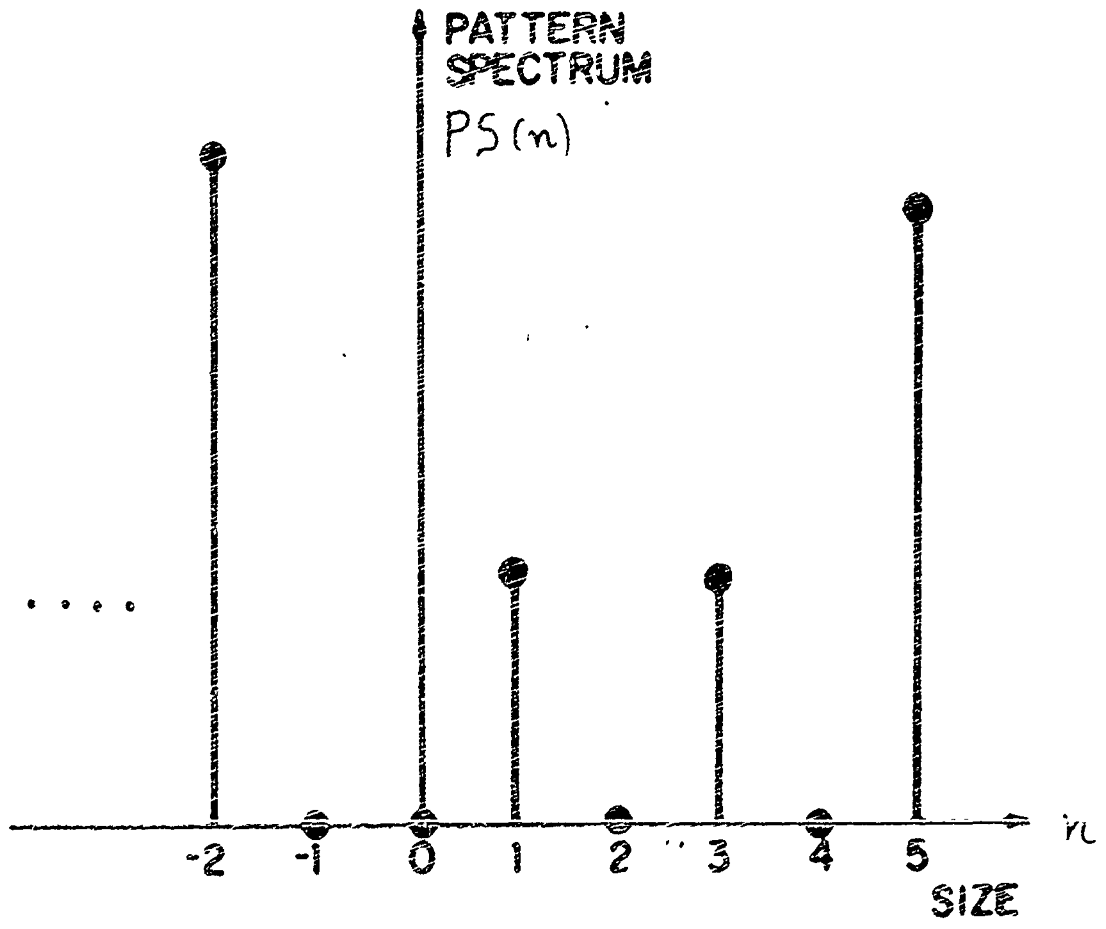




- SETS X : $$\boxed{P(r) = -\frac{dA\left(\bigcup_{\theta} X \circ rL_{\theta}\right)}{dr}, \quad r \geq 0}$$

- FUNCTIONS f : $$P(r) = -\frac{dA\left(\max_{\theta} f \circ rg_{\theta}\right)}{dr}, \quad r \geq 0$$

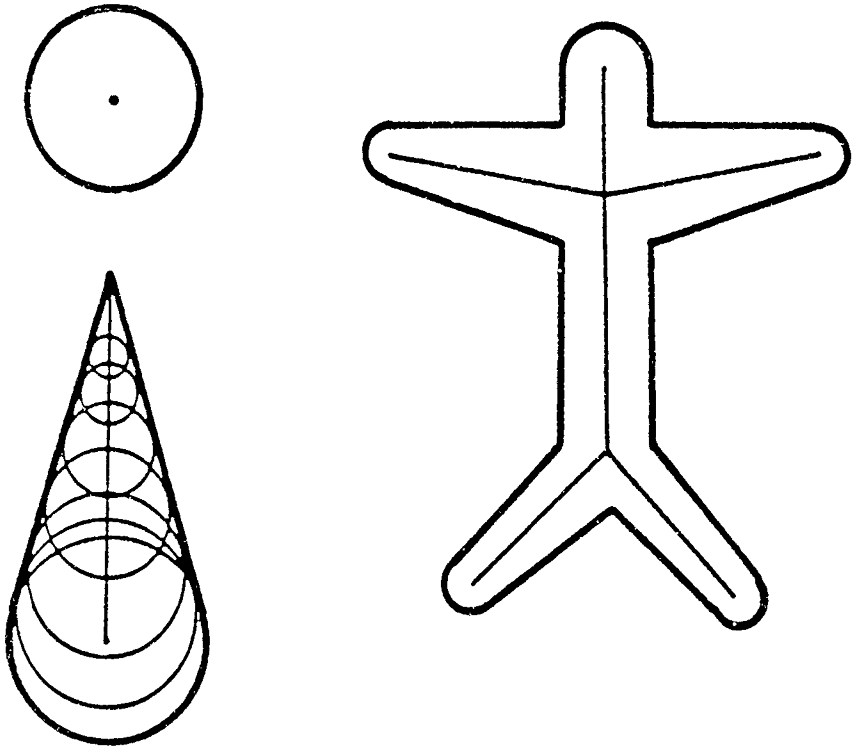


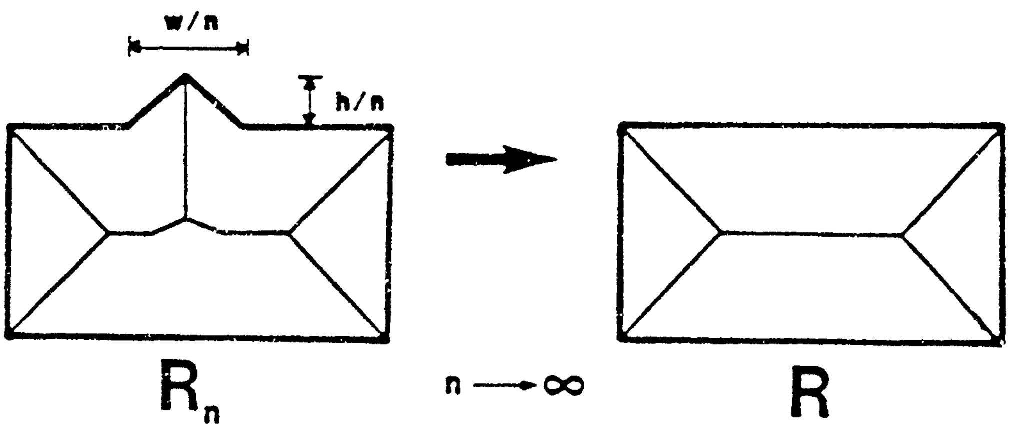


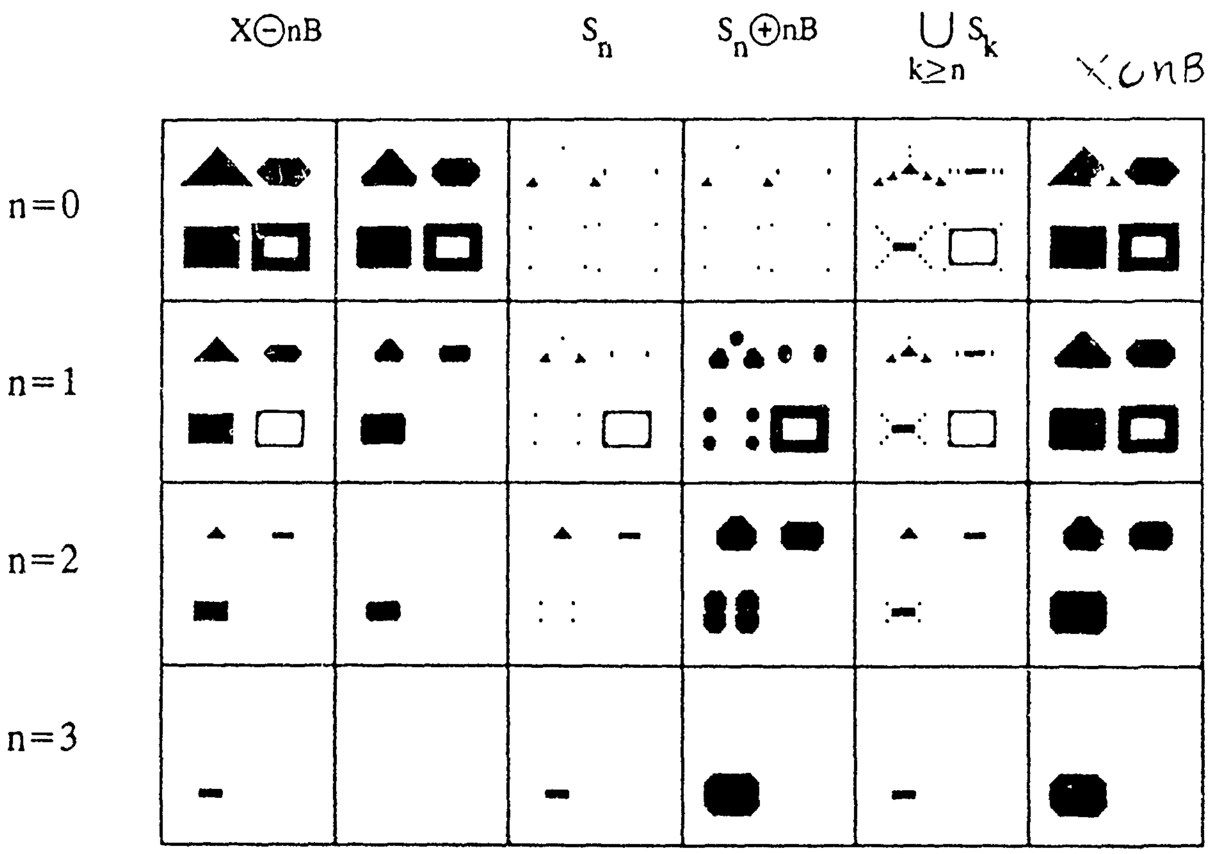
• SHAPE-SIZE COMPLEXITY MEASURE:

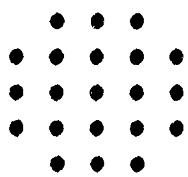
$$\boxed{H(f/g) = -\sum_{n} P_n \log(P_n)}\ , \quad P_n = \frac{PS(n,g)}{A(f)}$$

SKELETONS









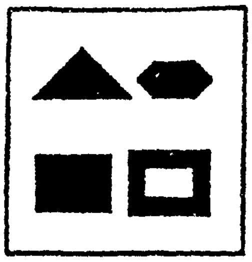
$= B$

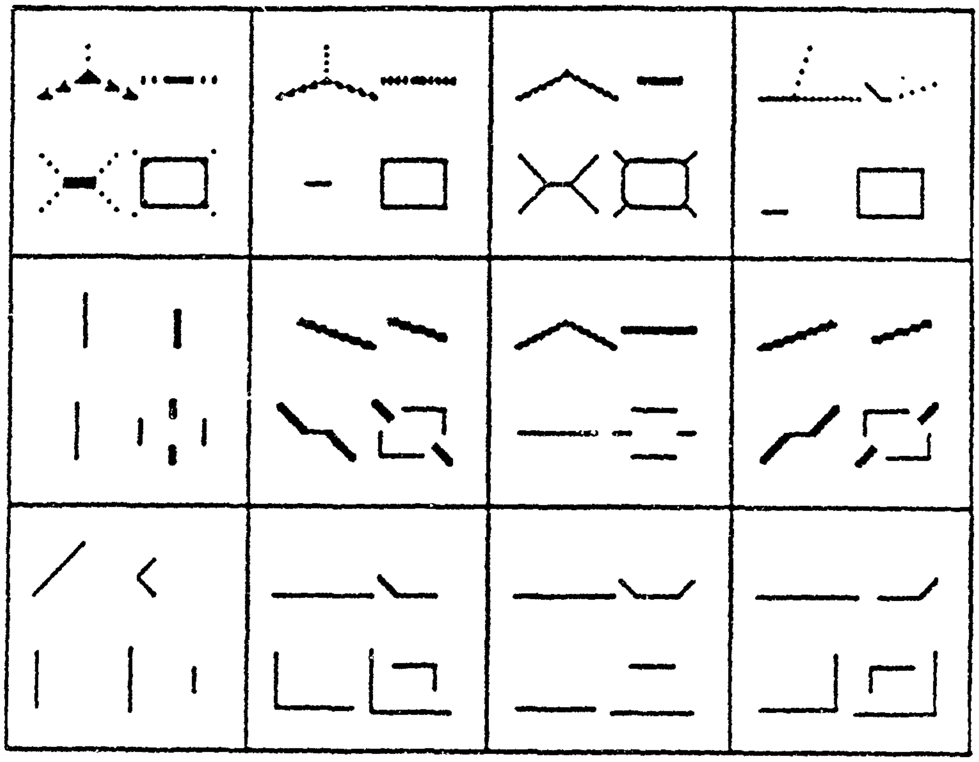
$$nB = B \oplus B \oplus \ldots \oplus B$$

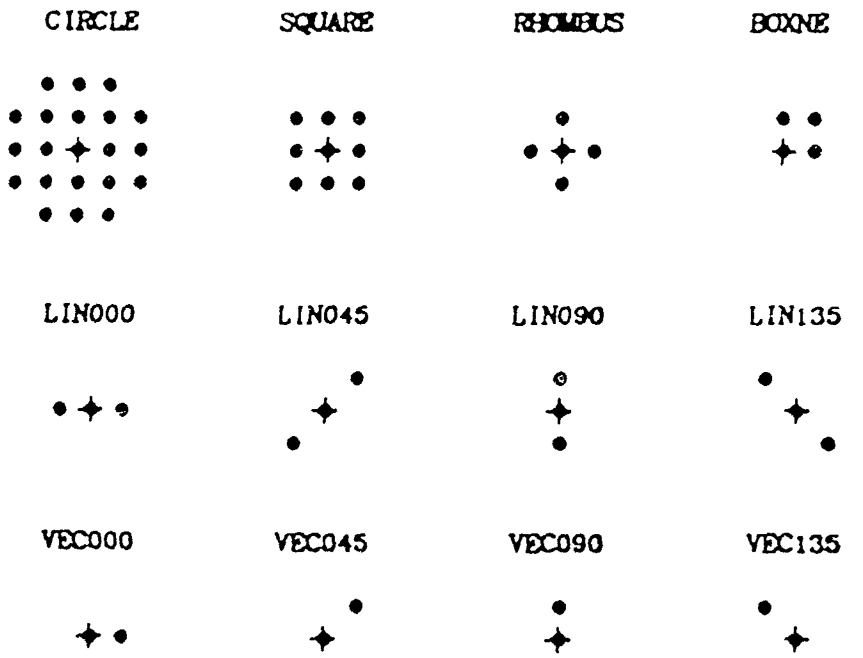
- skeleton subsets: $S_n = X \ominus nB - [(X \ominus nB) \circ B]$, $n = 0, 1, 2, \ldots, N$

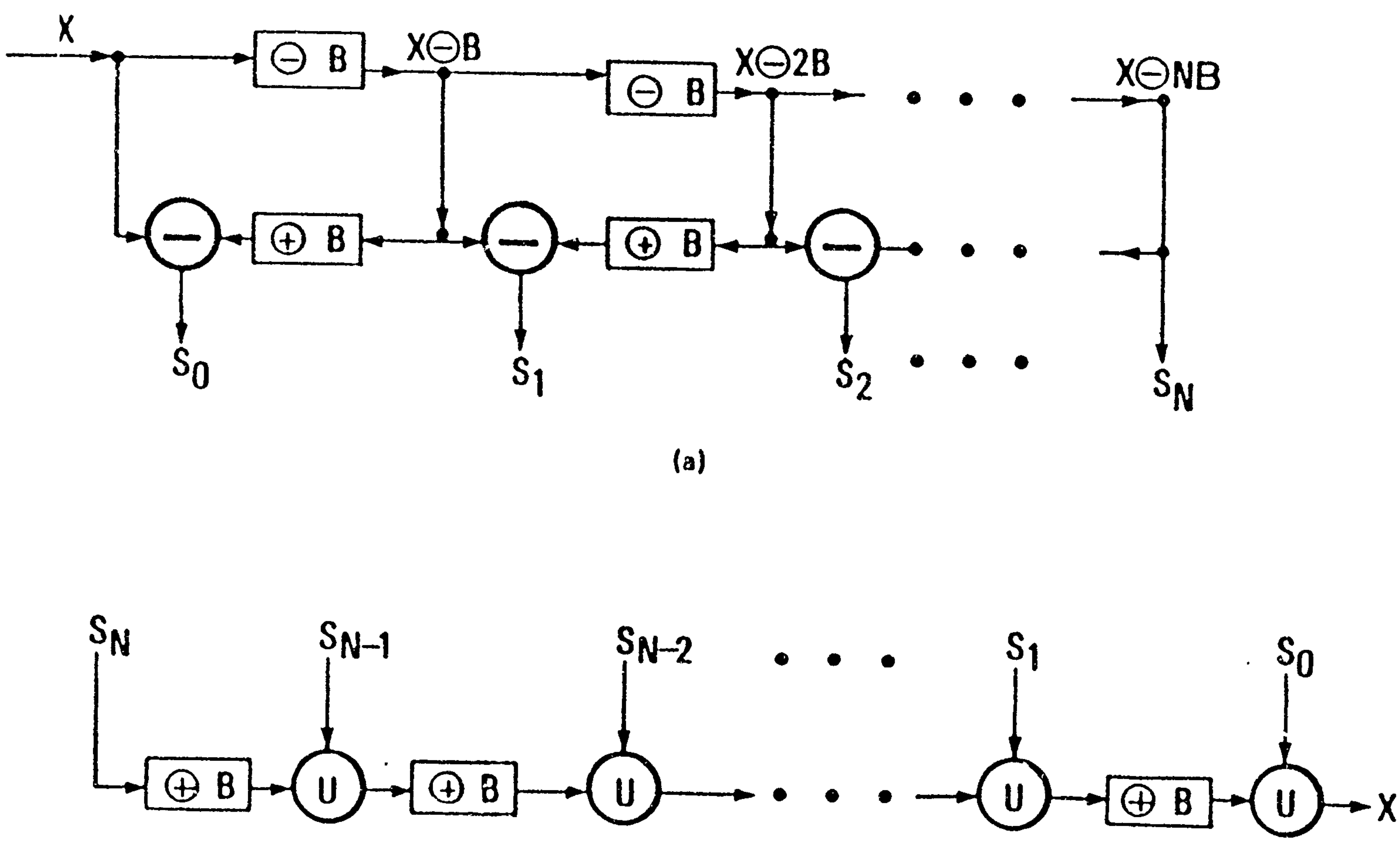
- skeleton: $SK(X) = \bigcup_{n=0}^{N} S_n$

- reconstruction: $\underbrace{X \circ kB}_{\text{multi-scale openings}} = \bigcup_{k \leq n \leq N} S_n \oplus nB$






CIRCLE
SQUARE
RHOMBUS
BOXNE
LIN000
LIN045
LIN090
LIN135
VEC000
VEC045
VEC090
VEC135



(a)

(b)

ORIGINAL IMAGES

ORIGINAL SKELETONS

MINIMAL SKELETONS

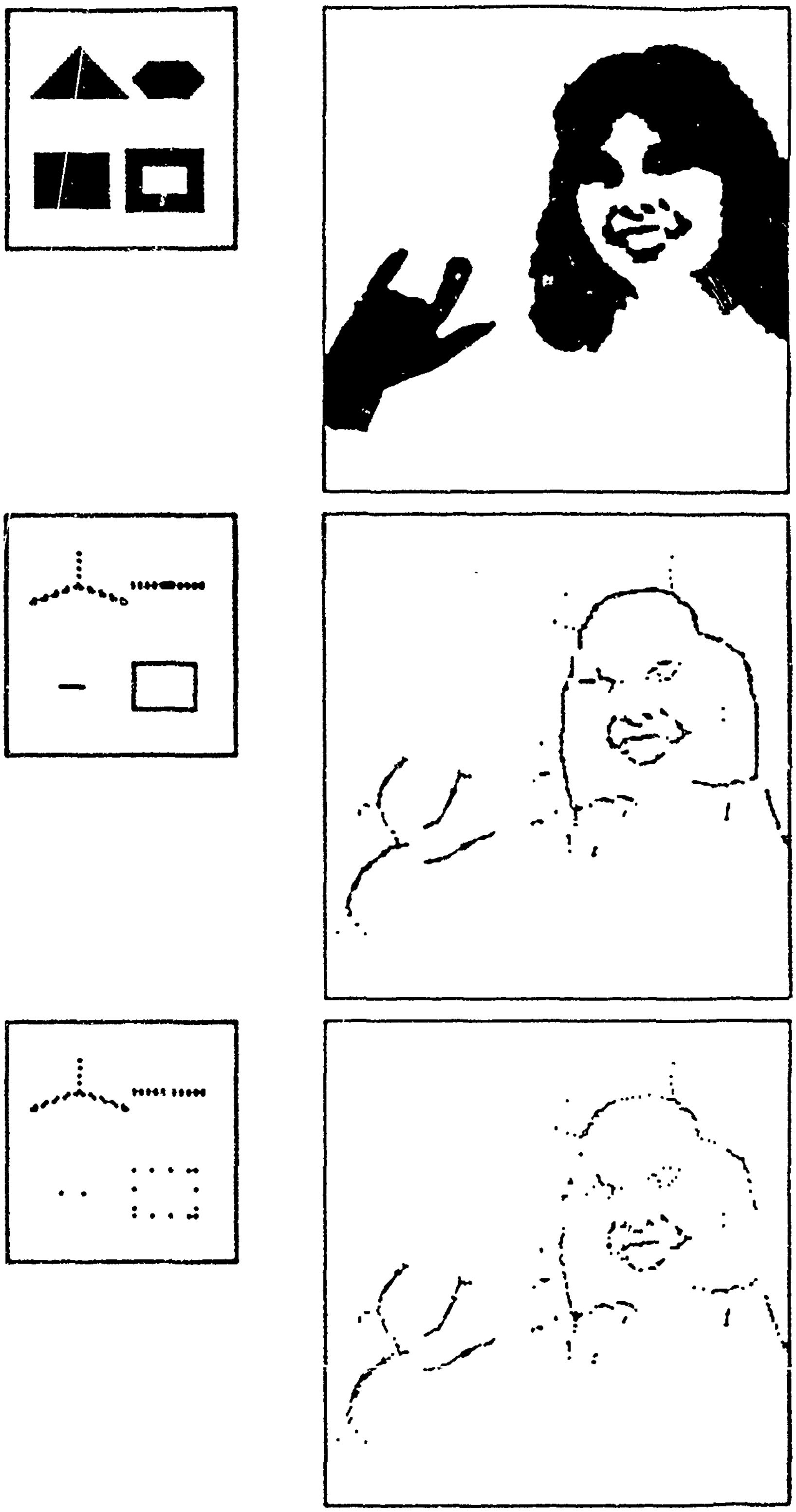


Figure 6-6. Images, skeletons, and globally minimal skeletons (struct. element=SQUARE).

SKELETON CODING OF BINARY IMAGES

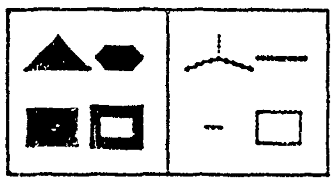
- SKELETON SUBSETS:

binary images $S_n(X)$ $n=0,1,2,\ldots,N$

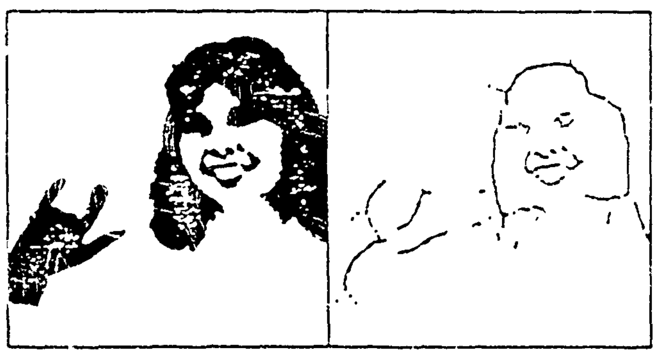
- SKELETON FUNCTION:

greytone image $[\,skf(X)\,](i,j) = \begin{cases} n+1, & (i,j) \in S_n(X) \\ 0, & (i,j) \notin SK(X) \end{cases}$

- COMPRESSION EFFICIENCY:

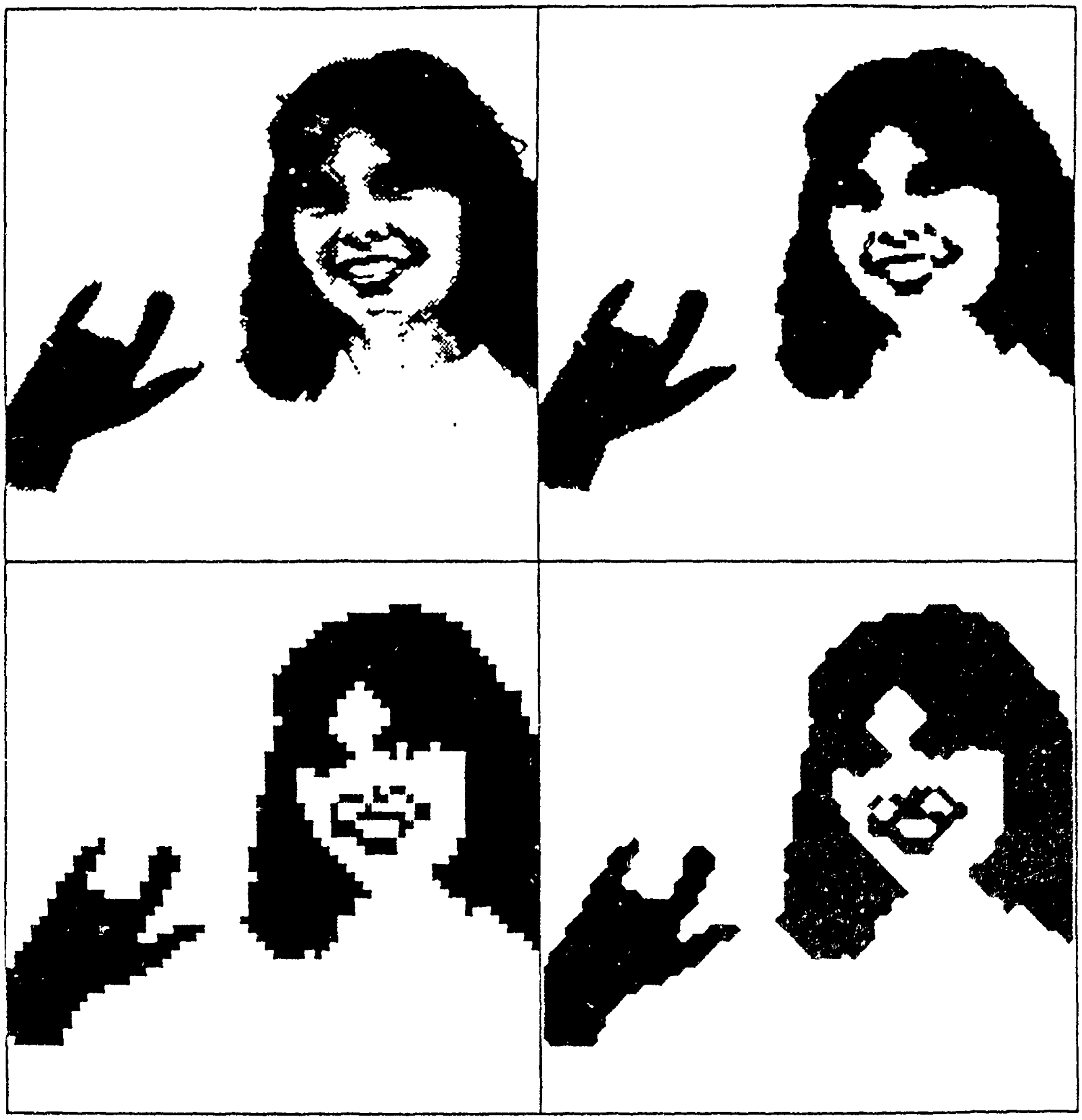


skeleton (Elias) = 8.0 , runlength (Huffman) = 4.9 , block (Huffman) = 4.3



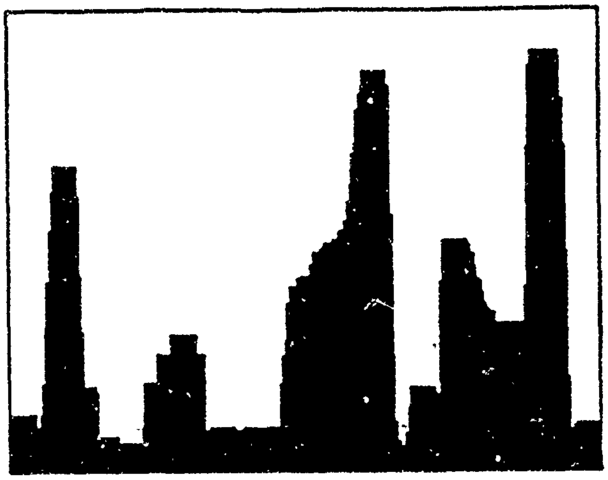
skeleton (Elias) = 11.4 , runlength (Huffman) = 8.3 , block (Huffman) = 5.1

(a) ORIGINAL 256×256 (b) OPENING



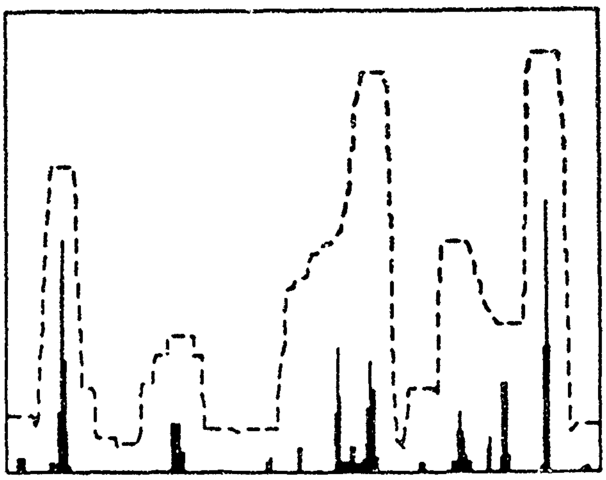
(c) DECIMATED 64×64 (d) CLOS - OPENING

ORIGINAL FUNCTION

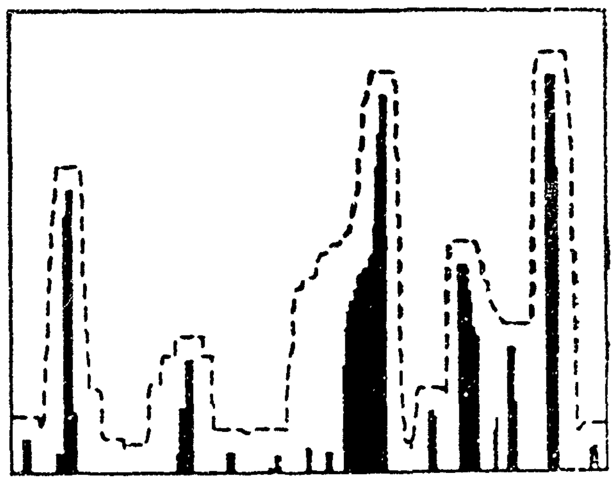


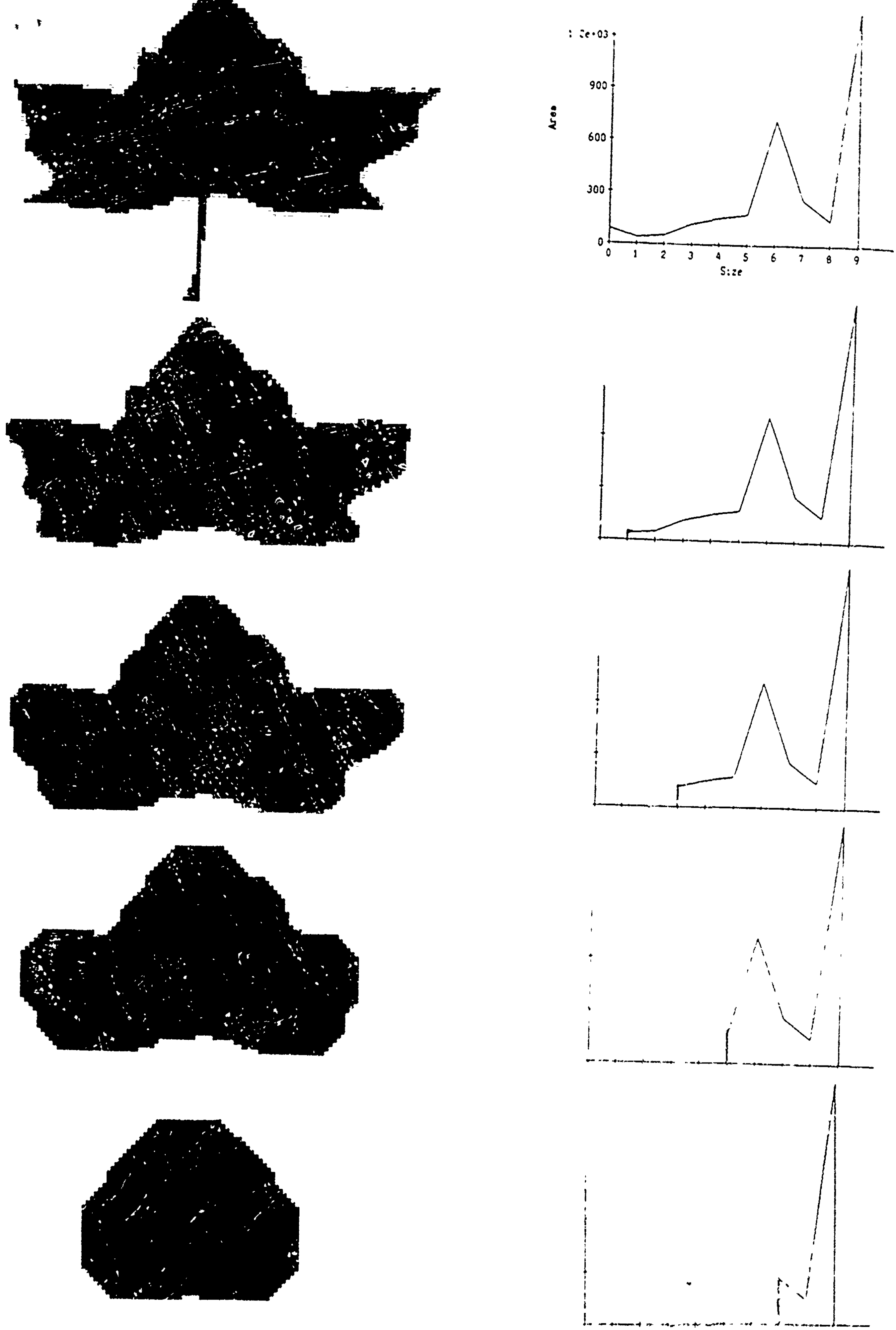
MAX OF ALGEBR. SKEL. SUBFUNCT.

SUM OF ALGEBR. SKEL. SUBFUNCT.

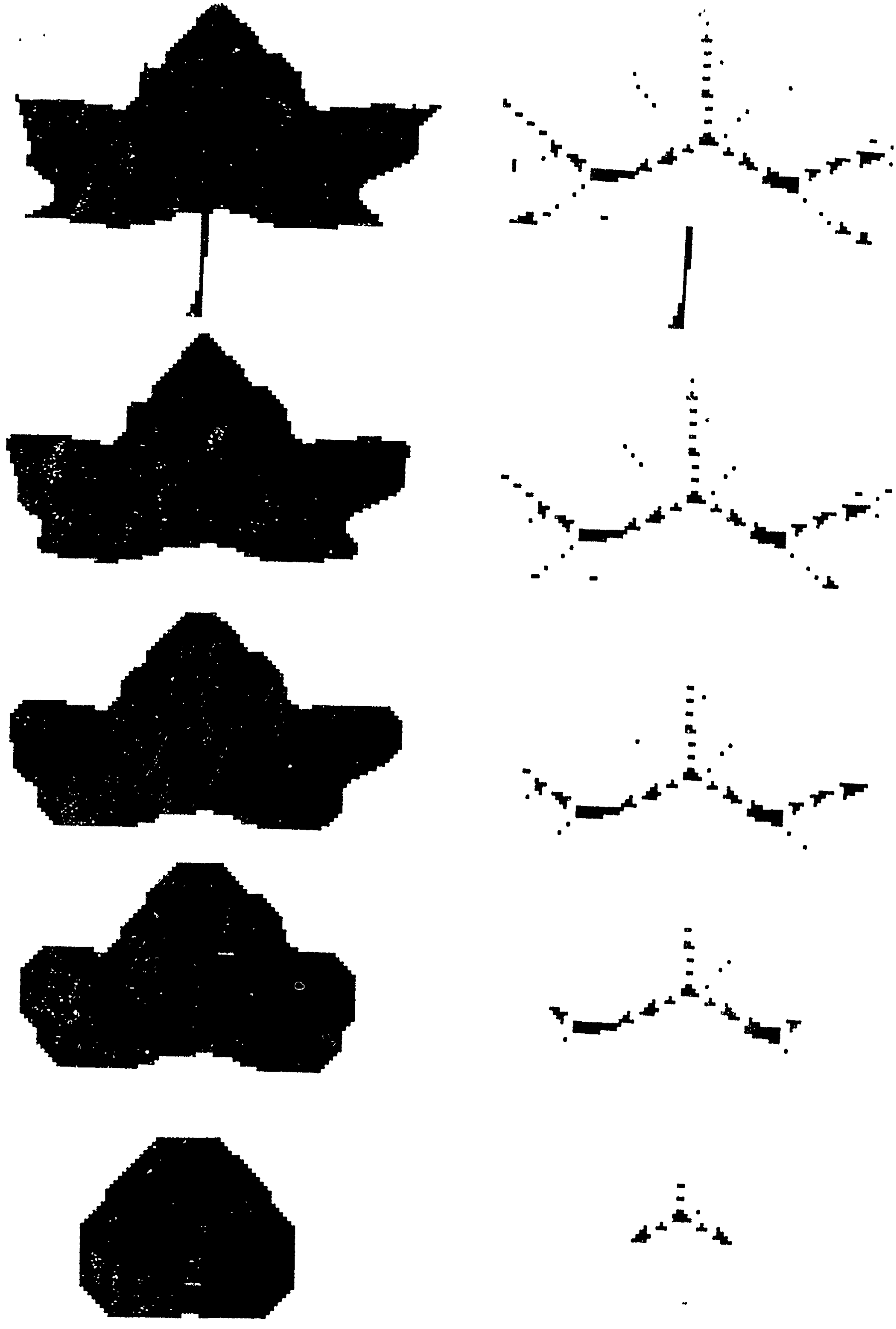


MAX OF MORPHOL. SKEL. SUBFUNCT.





Area
900
600
300
0
0
1
2
3
4
5
6
7
8
9
Size



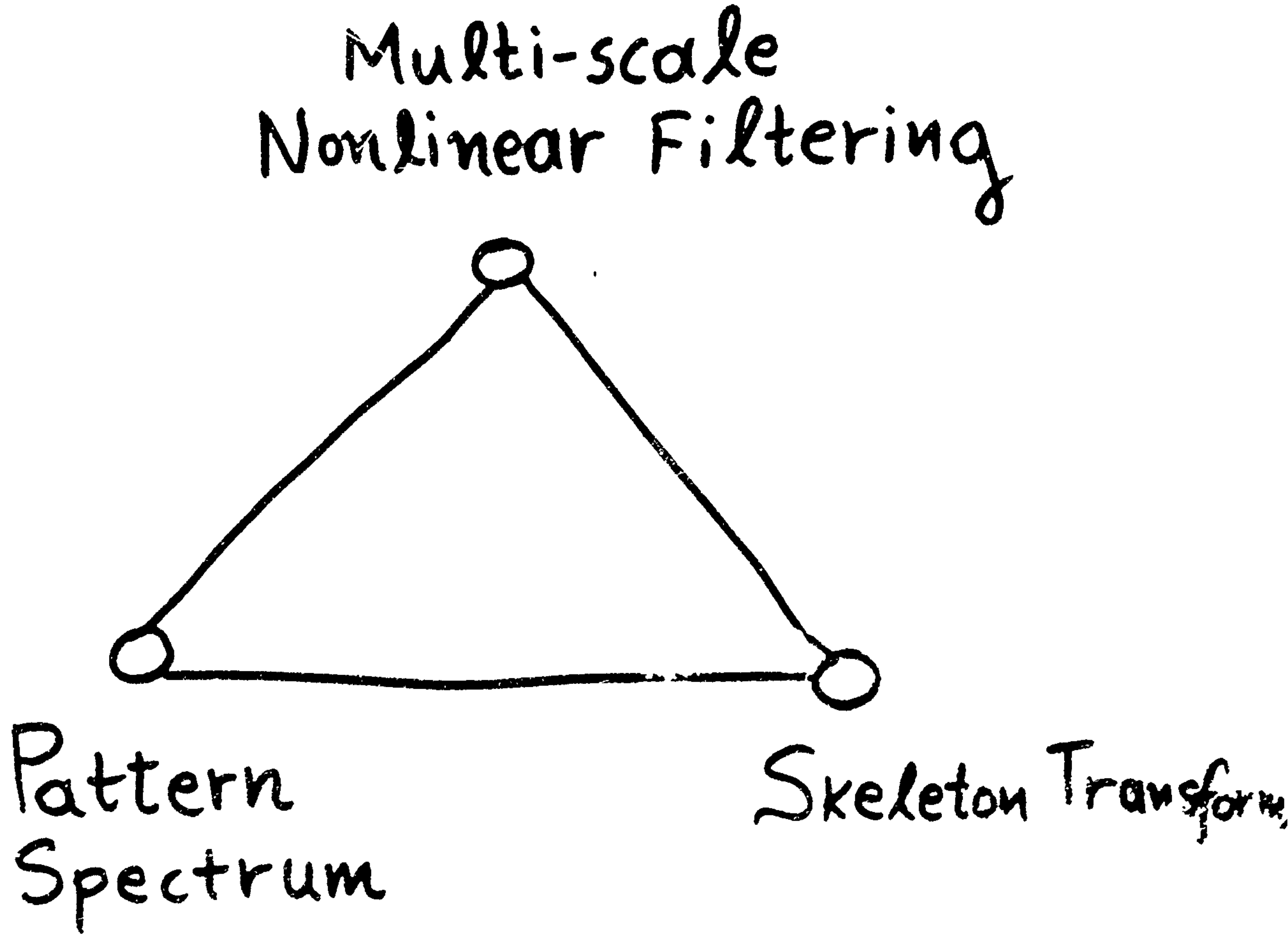
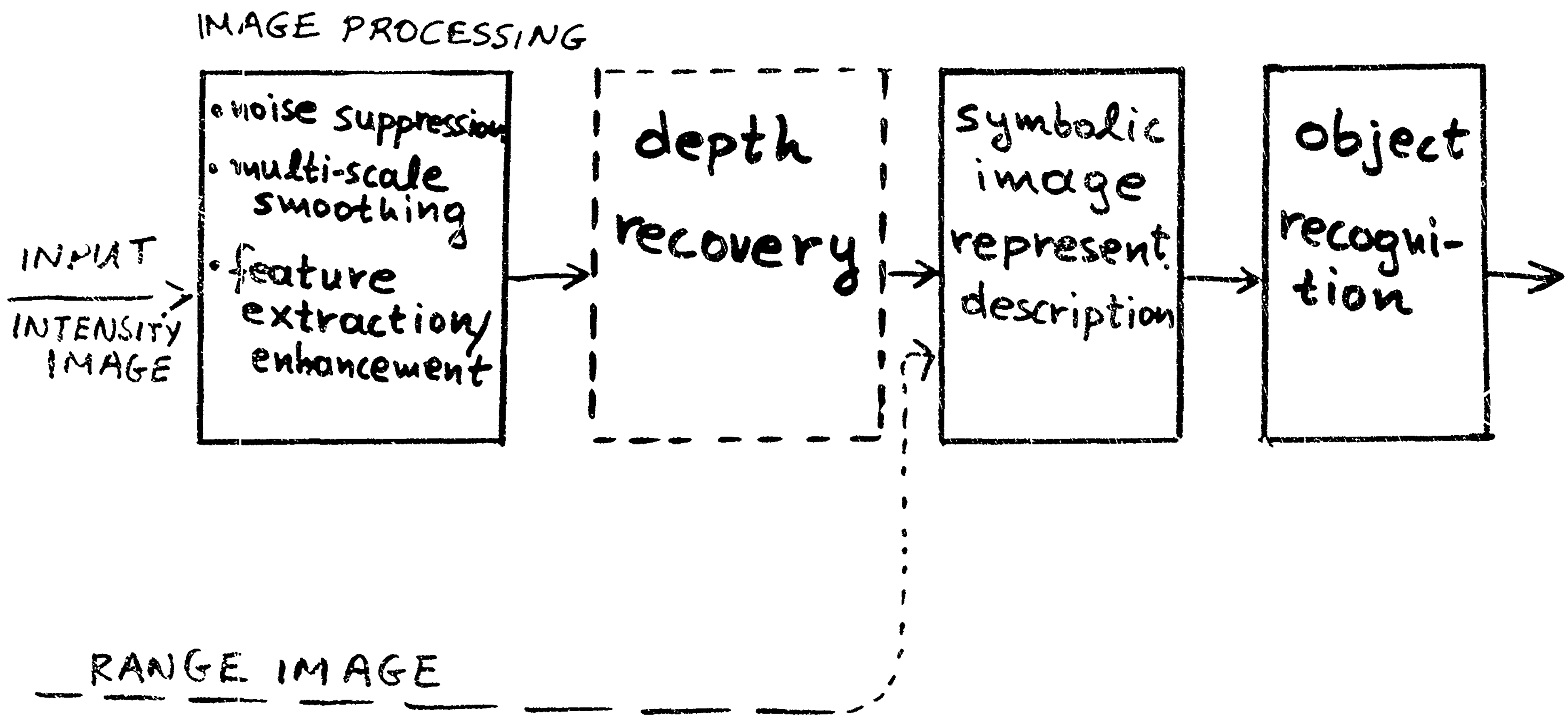
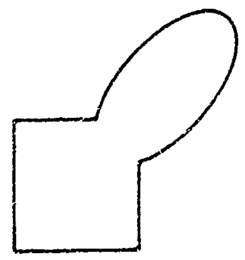
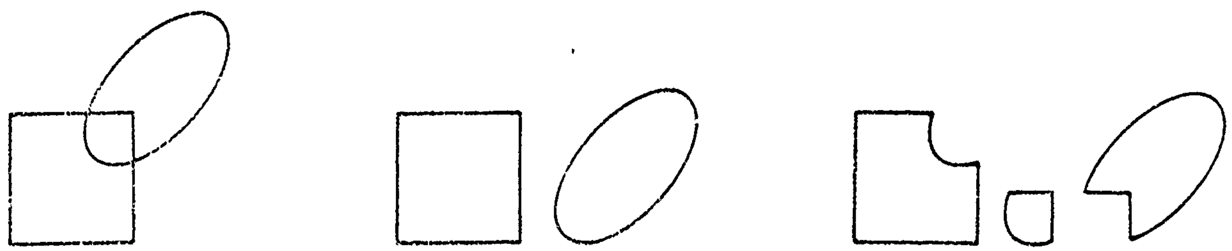

Multi-scale
Nonlinear Filtering
Pattern
Spectrum
Skeleton Transform

IMAGE ANALYSIS / UNDERSTANDING





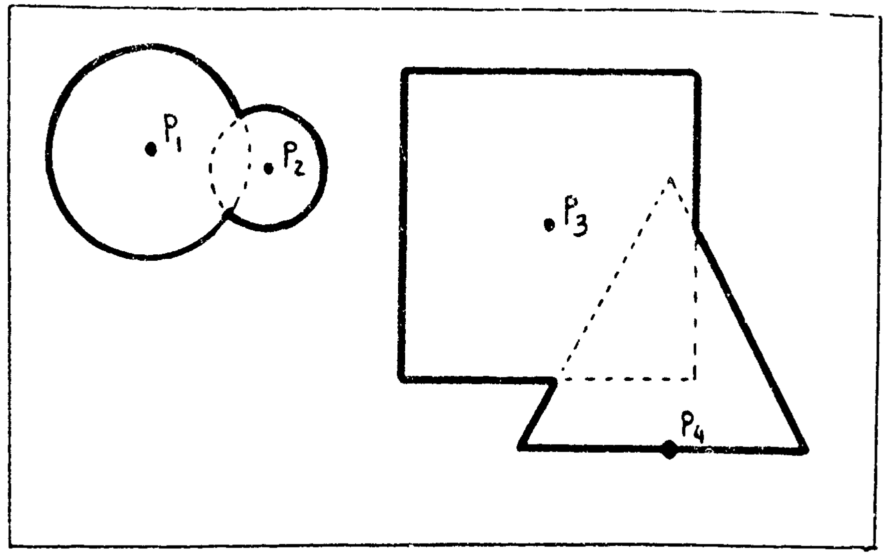


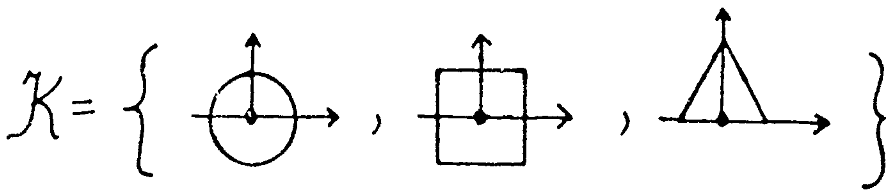
Model Parameters

- shapes
- sizes
- locations

MODEL:

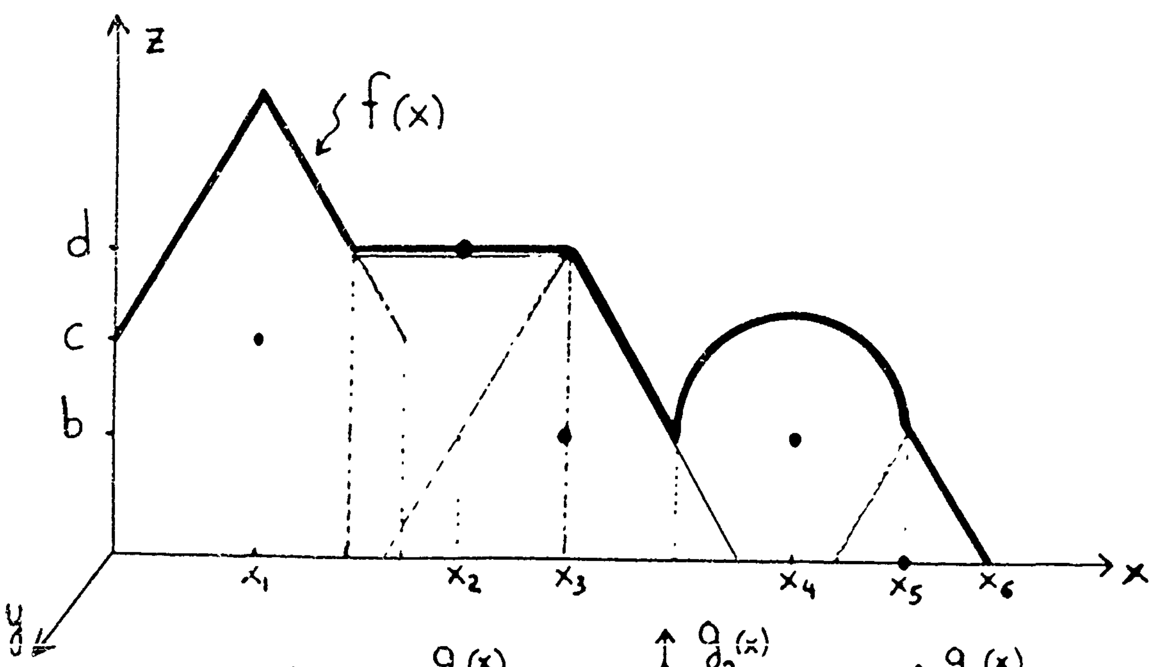
Minimal union of maximal shapes contained in the image

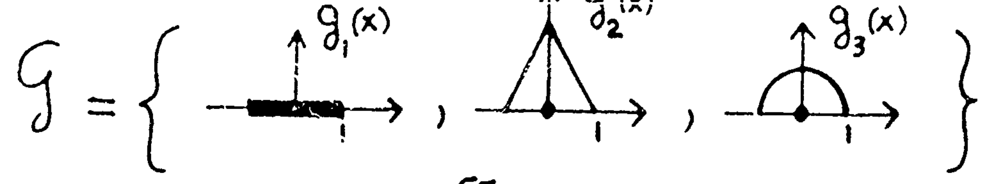




$$\boxed{X = \bigcup_i (n_i B_i) + P_i}$$

: minimal union of maximal patterns

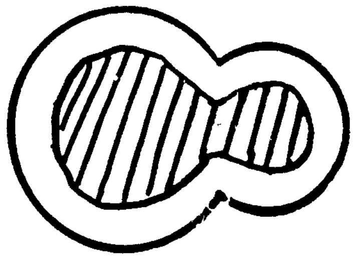




$$f(x) = \underset{1 \le i \le M}{\mathrm{MAX}} \ \underset{0 \le n \le N_i}{\mathrm{MAX}} \{[n g_i \oplus h_{n,i}](x)\}$$

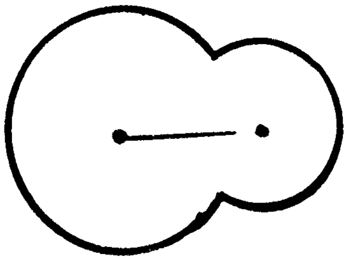
one shape pattern 

locations of shapes contained



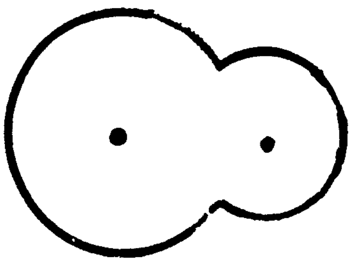
erosio

maximal shapes



skeleton

minimal #

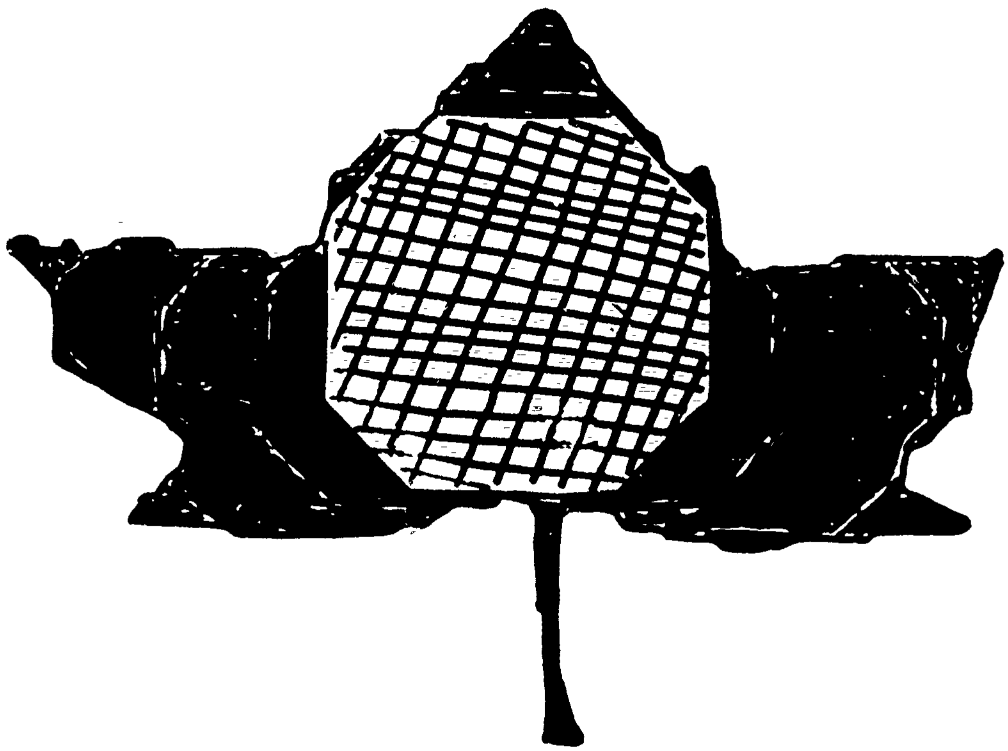


minimal skeleton

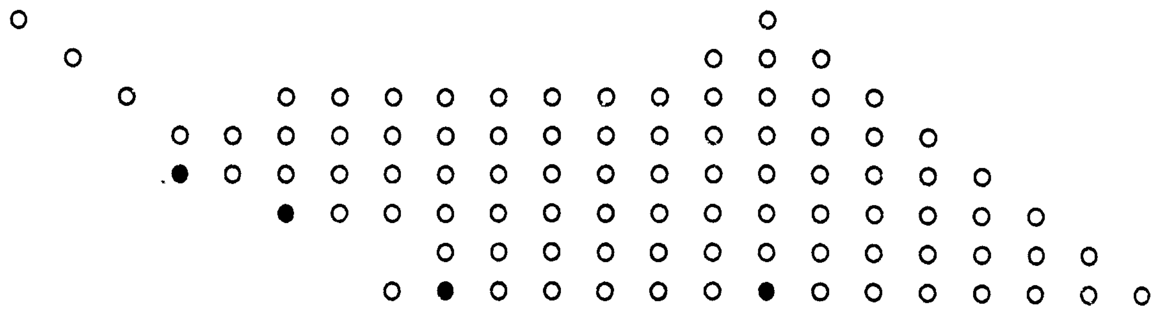
MULTIPLE SHAPE PATTERNS ?

multi-scale ⟶ nonlinear smoothing ⟶ OPEN...

critical scales ⟶ PATTERN SPECTRUM



complexity . (M·N)!

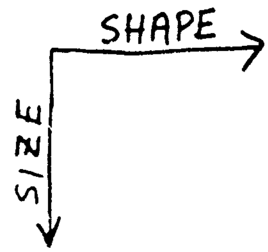


0	5	3	4	6
1	6	0	0	6
2	3	3	9	6
3	16	0	16	16
4	5	5	0	5
5	36	60	0	54
6	14	14	0	0
7	8	8	64	0

PATTERN SPECTRUM

0	2.55	1.67	1.33	1.88
1	2.38	1.51	1.12	1.65
2	2.17	1.51	1.12	1.38
3	2.01	1.35	0.72	1.08
4	1.61	1.35	0	0.42
5	1.32	1.09	0	0
6	0.95	0.95	0	0
7	0	0	0	0

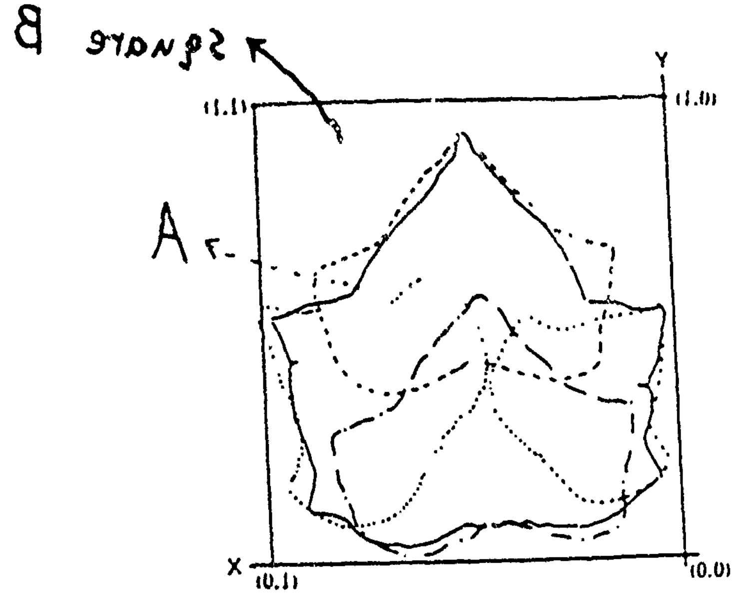
SHAPE-SIZE COMPLEXITY



i / n				
0	0	0	0	0
1	0	0	0	1
2	0	0	0	0
3	0	0	0	1
4	0	0	0	0
5	0	1	0	1
6	0	0	0	0
7	0	0	1	0

SHAPE-SIZE CONTAINMENT

ITERATIVE MODELING of FRACTAL-LIKE IMAGES



COLLAGE THEOREM (M. BARNSLEY)

$$w_i(z) = s_i z + t_i \;,\; 0 \le |s_i| \le s < 1 \;,\; i = 1, 2, \dots, N$$

$$W(A) = \bigcup_i w_i(A) .$$

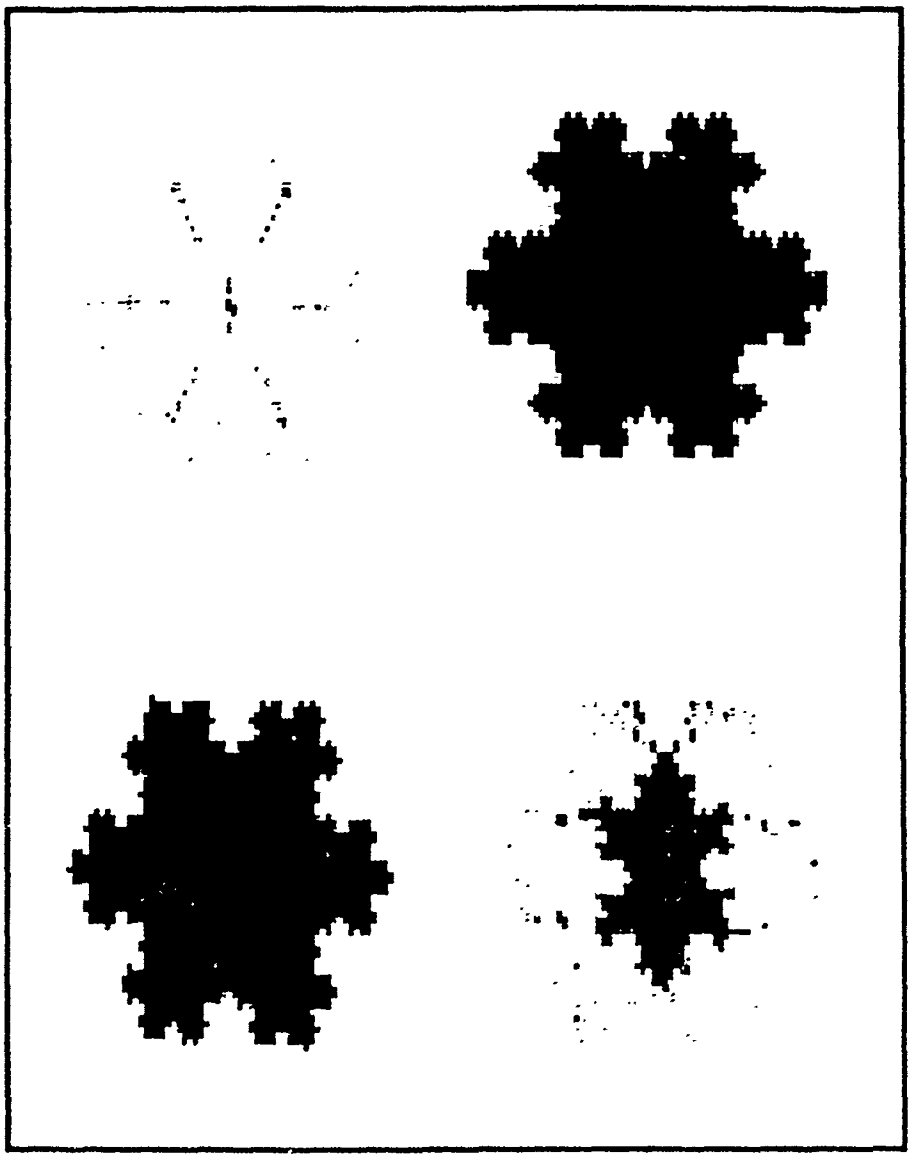
$$\boxed{h\Big(A, \bigcup_i w_i(A)\Big) < \varepsilon \Longrightarrow h\Big(A, \lim_{n\to\infty} W^{on}(B)\Big) < \varepsilon/(1-s)}$$

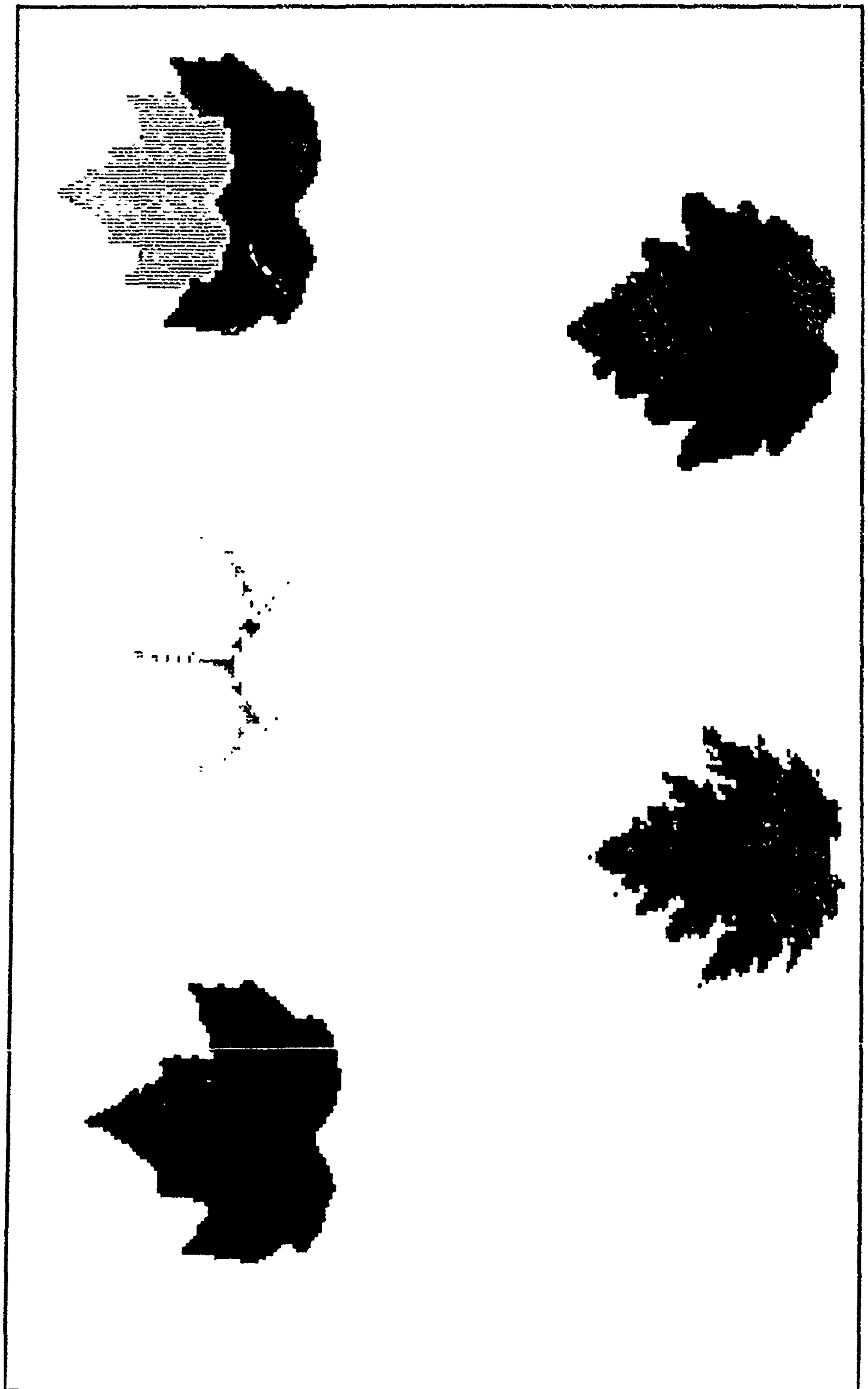
UNIQUE ATTRACTOR

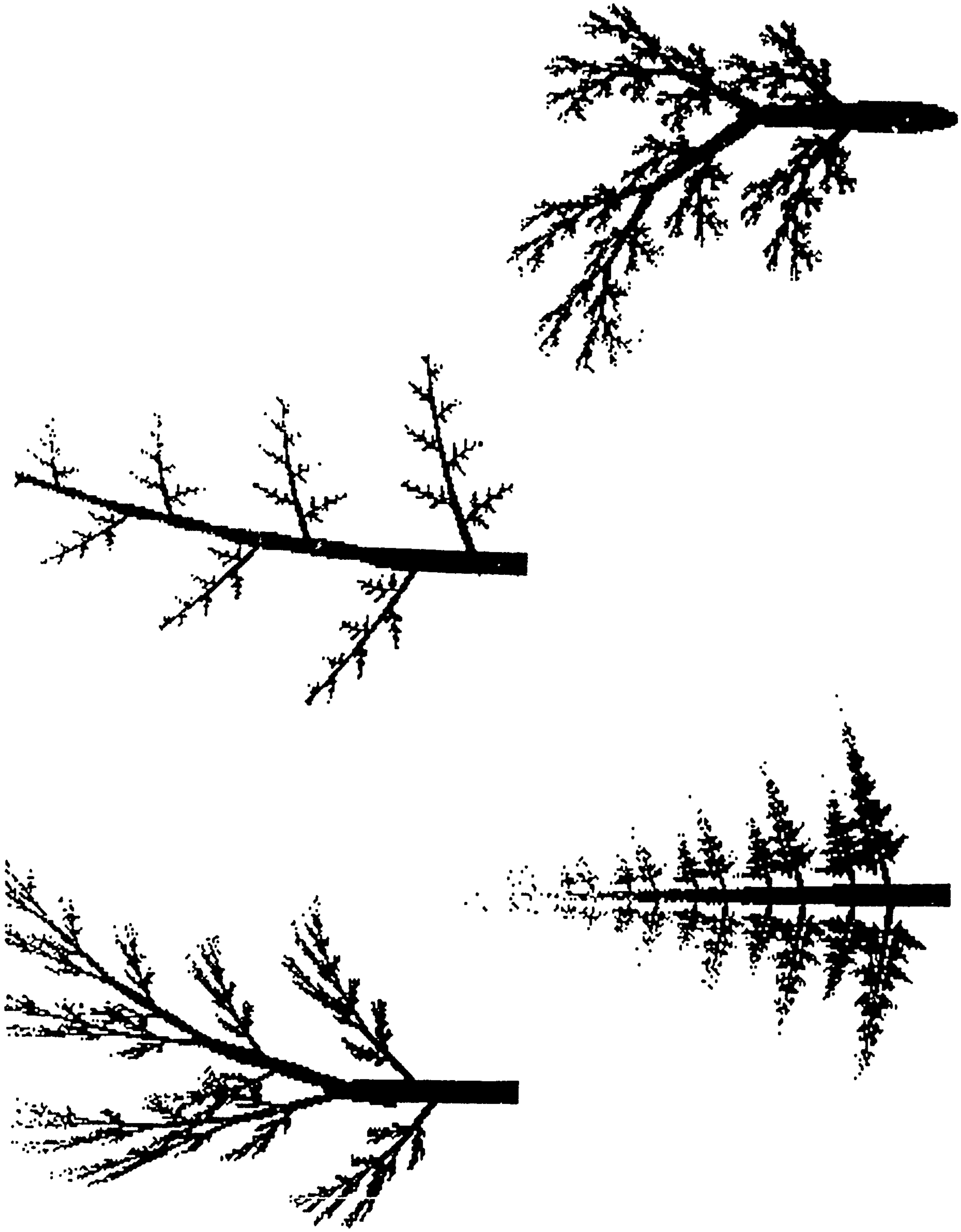
* PROBLEM : Find w_i

* Approach :

$$w_i \begin{bmatrix} x \\ y \end{bmatrix} = \underset{\text{scaling}}{r_i} \underbrace{\begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix}}_{\text{rotation}} \begin{pmatrix} x \\ y \end{pmatrix} + \underset{\text{translation}}{\begin{bmatrix} b_{ix} \\ b_{iy} \end{bmatrix}}$$







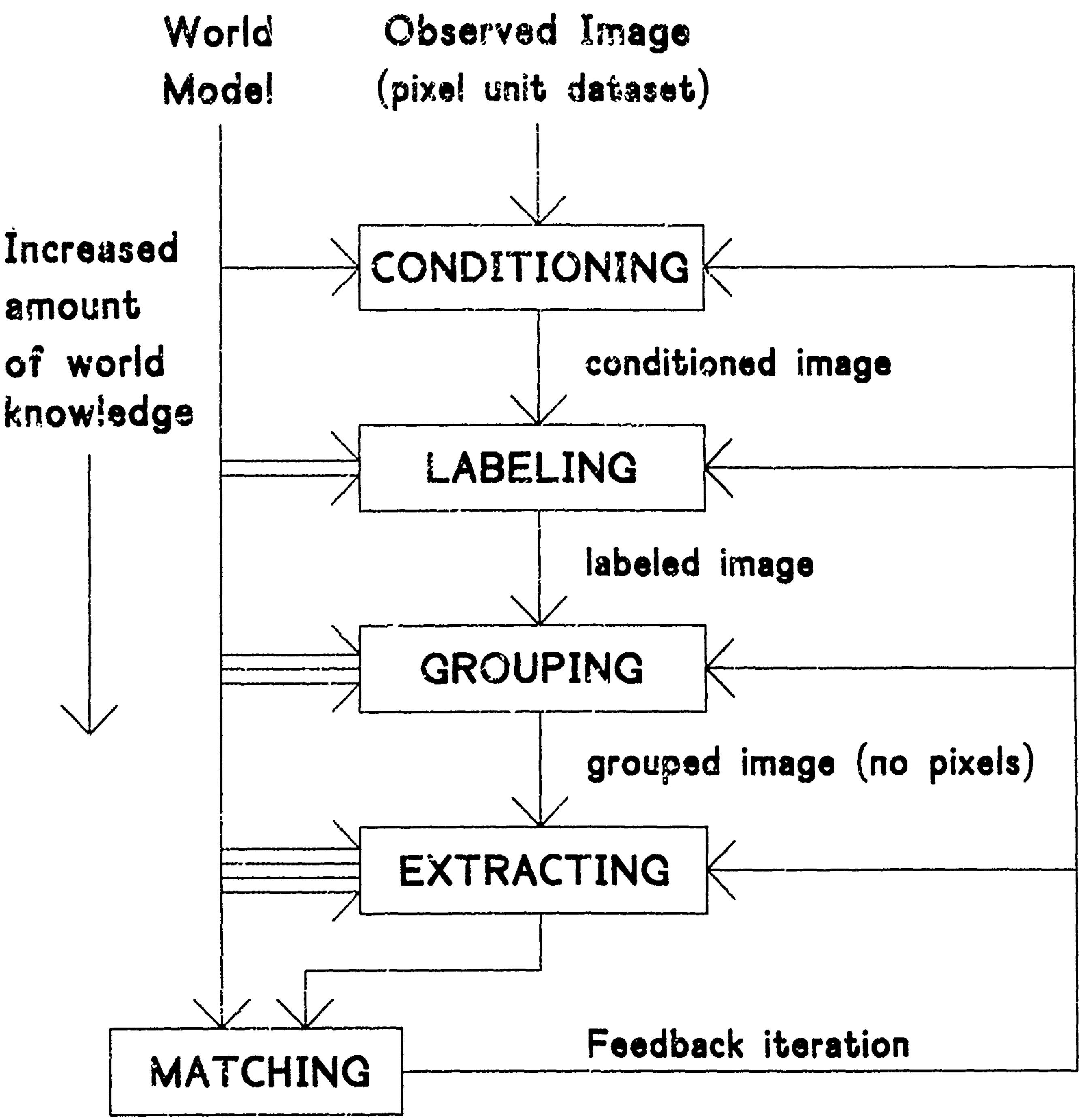
Cont 8 ARO 26131-1-EL-CF

Mathematical Morphology Overview

Robert M. Haralick

Intelligent Systems Laboratory
Department of Electrical Engineering • FT-10
University of Washington
Seattle, WA 98195

Taxonomy for Computer Vision



Interpretation of Observed Image

MATHEMATICAL MORPHOLOGY

DILATION

TRANSLATION

REFLECTION

WAYS OF UNDERSTANDING OR
CHARACTERIZING DILATION

PROPERTIES OF DILATION

EROSION

WAYS OF CHARACTERIZING EROSION

PROPERTIES OF EROSION

OPENING

CLOSING

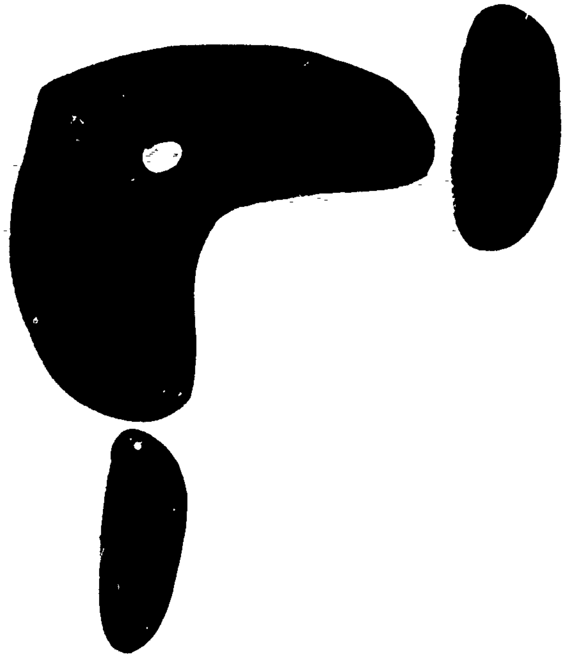
DUALITY

PROPERTIES OF OPENING AND CLOSING

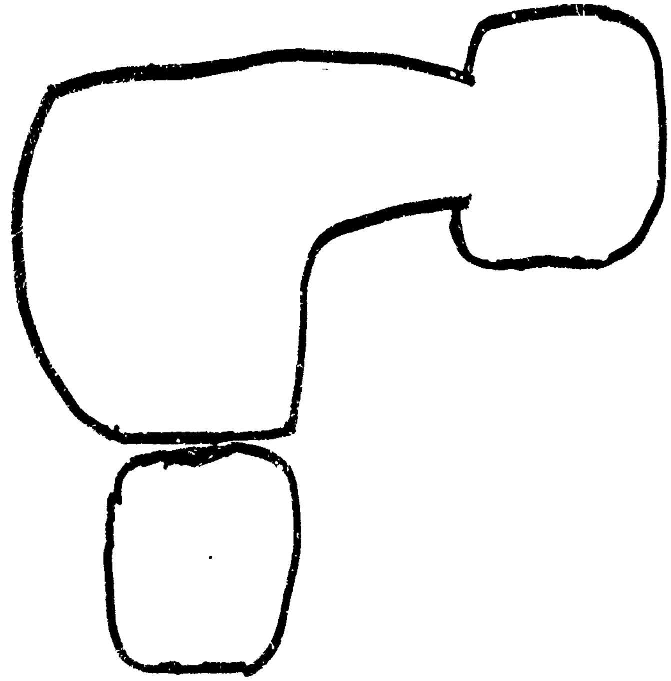
UMBRA

PROPERTIES OF UMBRA

GRAY TONE MORPHOLOGY



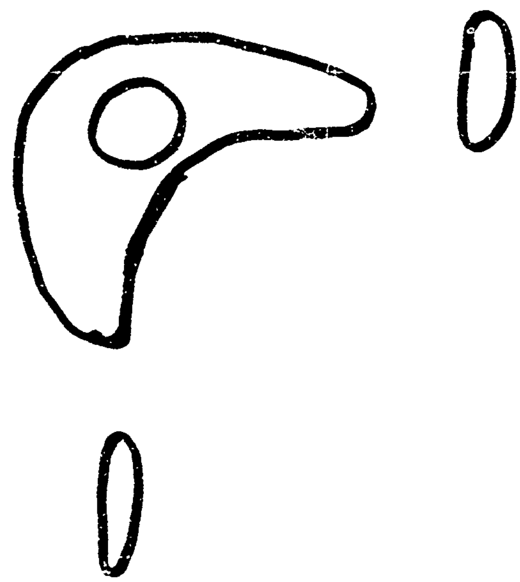
ORIGIN



DILATION

RUN ORIGIN OF THE STRUCTURING ELEMENT OVER ALL THE BINARY ONE PIXELS OF THE IMAGE. THE AREA SWEPT BY THE STRUCTURING ELEMENT IS THE DILATED IMAGE

ORIGIN OF STRUCTURING ELEMENT
IS ITS CENTER

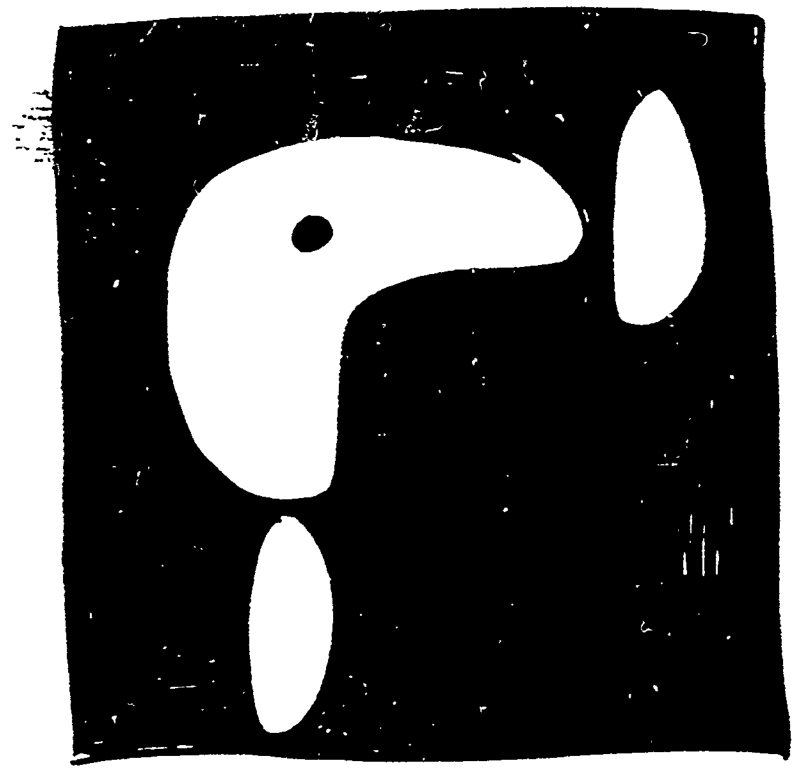


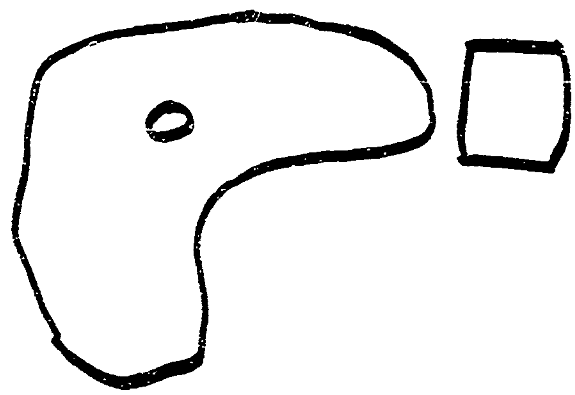
EROSION

RUN THE ORIGIN OF THE STRUCTURING ELEMENT OVER ALL PIXELS OF THE IMAGE. MARK THOSE PIXELS AT WHICH THE STRUCTURING ELEMENT ORIGIN CAN STAND AND WHERE ITS AREA ONLY COVERS BINARY ONE PIXELS. THE AREA OF MARKED PIXELS IS THE ERODED IMAGE.

DUALITY

WHAT DILATION DOES TO THE FOREGROUND EROSION DOES TO THE BACKGROUND





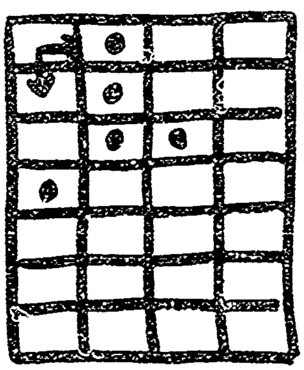
OPENING

RUN THE STRUCTURING ELEMENT THROUGH THE AREA OF BINARY ONE PIXELS KEEPING THE STRUCTURING ELEMENT'S AREA CONTAINED IN THE AREA OF BINARY ONE PIXELS. THE AREA SWEPT BY THE STRUCTURING ELEMENT IS THE OPENED IMAGE

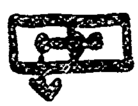
DILATION

DEF: LET A AND B BE SUBSETS OF E^N. THE DILATION OF A BY B IS DENOTED BY $A \oplus B$ AND IS DEFINED BY

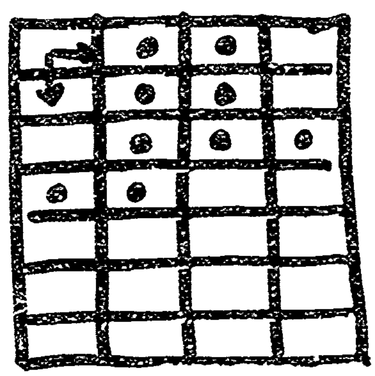
$$A \oplus B = \{ c \in E^N \mid c = a + b \text{ FOR SOME } a \in A \text{ AND } b \in B \}$$



A



B

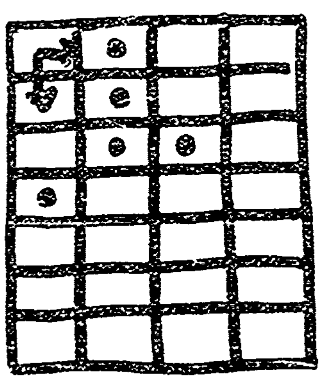


$A \oplus B$

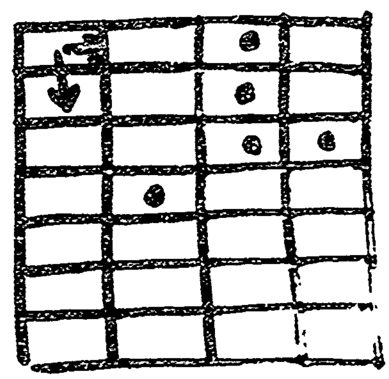
TRANSLATION

DEF: LET A BE A SUBSET OF E^n AND $x \in E^n$. THE TRANSLATION OF A BY x IS DENOTED BY A_x AND IS DEFINED BY

$$A_x = \{ c \in E^n \mid c = a + x \text{ FOR SOME } a \in A \}$$



A



$A_{(0,1)}$

PROPERTIES

$$A \oplus B = \bigcup_{a \in A} B_a$$

$$A \oplus B = \bigcup_{b \in B} A_b$$

$$A \oplus B = B \oplus A$$

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

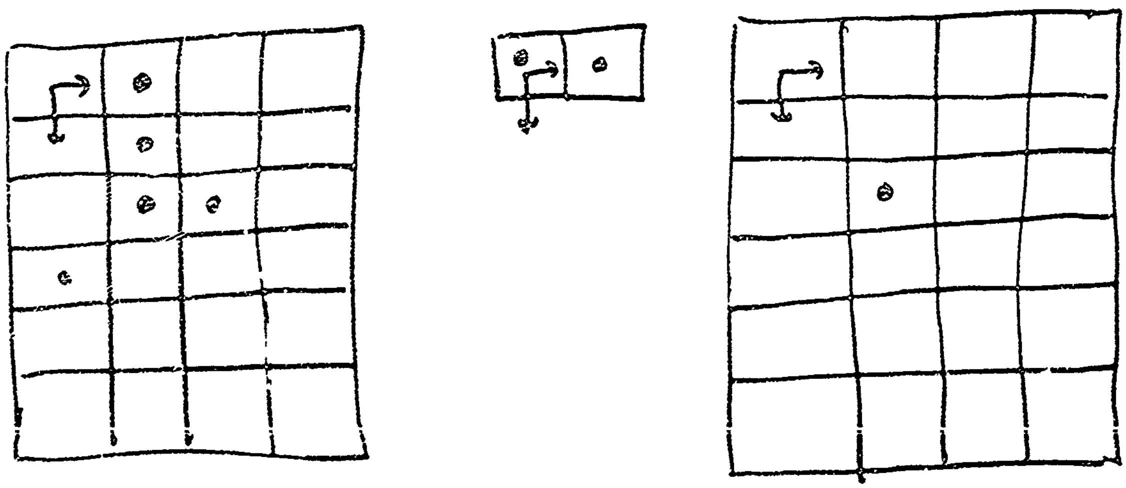
$A \subseteq B$ IMPLIES $A \oplus K \subseteq B \oplus K$

$$(A \cup B) \oplus K = (A \oplus K) \cup (B \oplus K)$$

EROSION

DEF: LET A AND B BE SUBSETS OF E^N. THE EROSION OF A BY B IS DENOTED BY $A \ominus B$ AND IS DEFINED BY

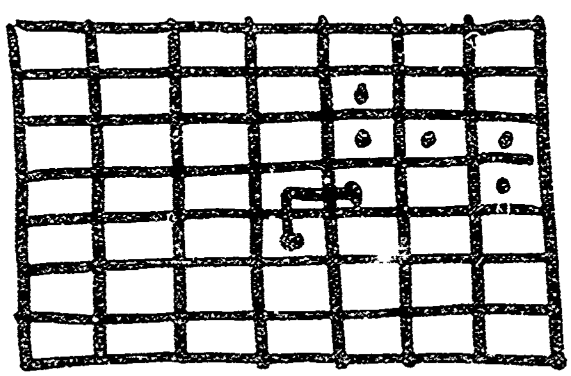
$$A \ominus B = \{x \in E^N \mid x + b \in A \text{ for every } b \in B\}$$



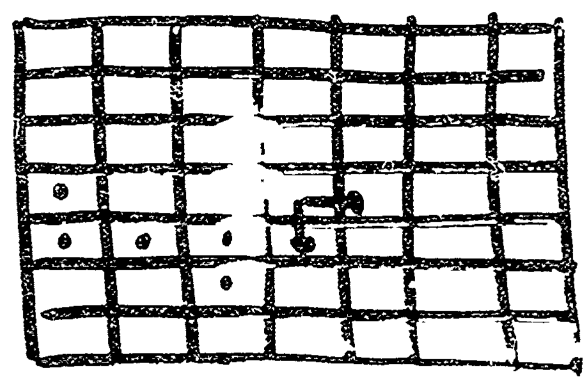
REFLECTION

DEF LET B BE A SUBSET OF E^n. THE REFLECTION OF B IS DENOTED BY $\check{B}$ AND IS DEFINED BY

$$\check{B} = \{x \mid x = -b \text{ FOR SOME } b \in B\}$$



B



$\check{B}$

PROPERTIES

$$A \ominus \text{[illegible]} \bigcap_{b \in B} A_{-b}$$

$A \subseteq B$ IMPLIES $A \ominus K \subseteq B \ominus K$

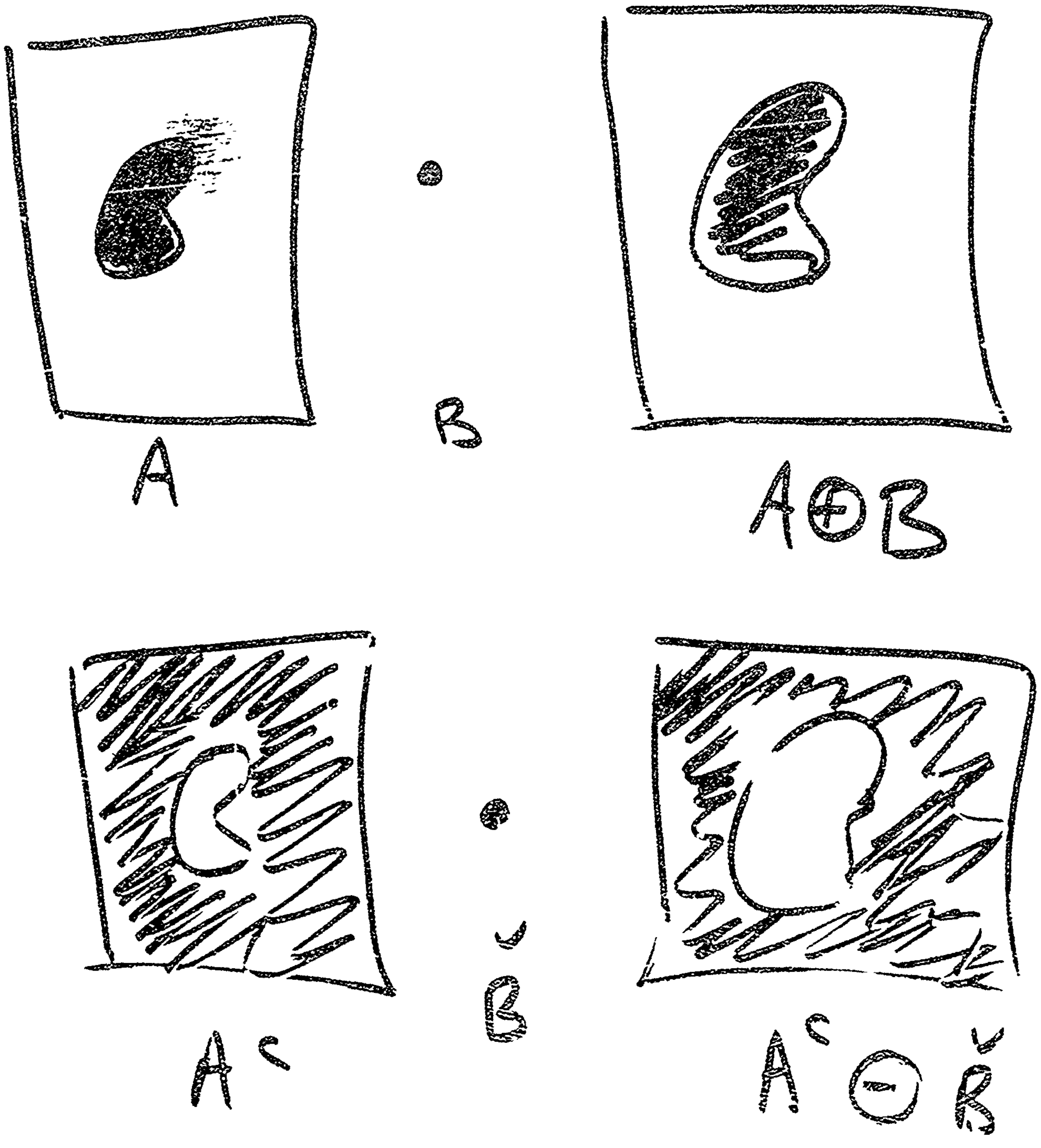
$$(A \cap B) \ominus K = (A \ominus K) \cap (B \ominus K)$$

$$A \ominus (B \cup C) = (A \ominus B) \cap (A \ominus C)$$

$$A \ominus (B \oplus C) = (A \ominus B) \ominus C$$

DUALITY

$$(A \ominus B)^c = A^c \oplus \check{B}$$


A
B
A⊕B
A^c
B̌
A^c⊖B̌

OPENING AND CLOSING

DEF: THE OPENING OF A BY B IS DENOTED BY $A \circ B$ AND IS DEFINED BY

$$A \circ B = (A \ominus B) \oplus B$$

DEF: THE CLOSING OF A BY B IS DENOTED BY $A \bullet B$ AND IS DEFINED BY

$$A \bullet B = (A \oplus B) \ominus B$$

PROPERTIES

$$A \circ \text{[illegible]} \subseteq A$$

$$A \bullet B \supseteq A$$

$$(A \bullet B)^c = A^c \circ \check{B}$$

$$A \circ B = \{ x \in A \mid \text{FOR SOME } y,\ x \in B_y \subseteq A \}$$

$$(A \circ K) \circ K = A \circ K$$

$$(A \bullet K) \bullet K = A \bullet K$$

IF $A \circ B = A$, THEN $(D \circ A) \circ B = D \circ A$

CLOSING & OPENING

Closed by B   Opened by B

CLOSING

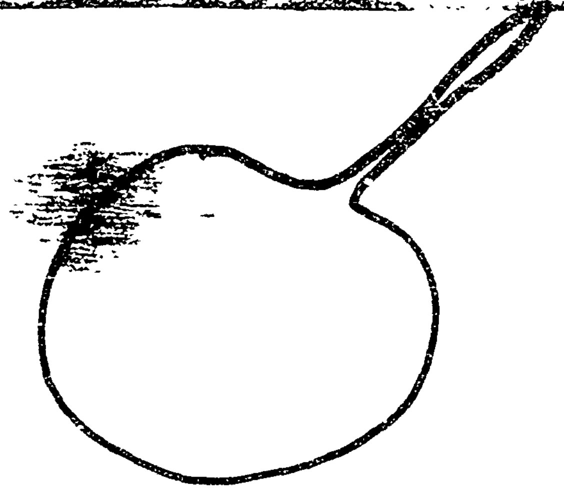
$A \bullet B = (X \oplus B) \ominus B$

complement/union of Disks contained in X^c.

OPENING

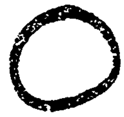
$A \circ B \quad (X \ominus B) \oplus B$

union of disks contained in X.

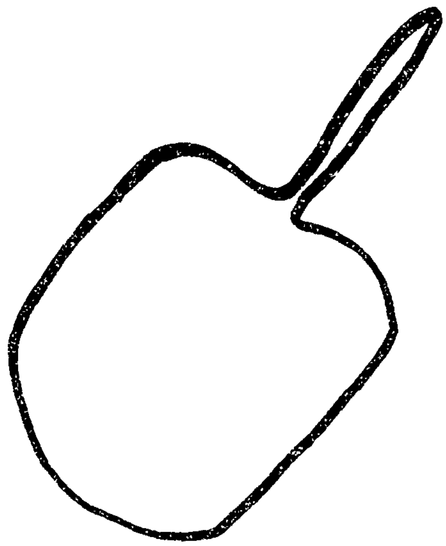


F

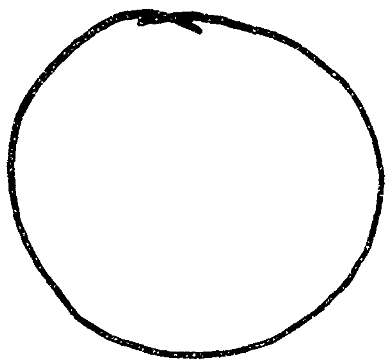
K



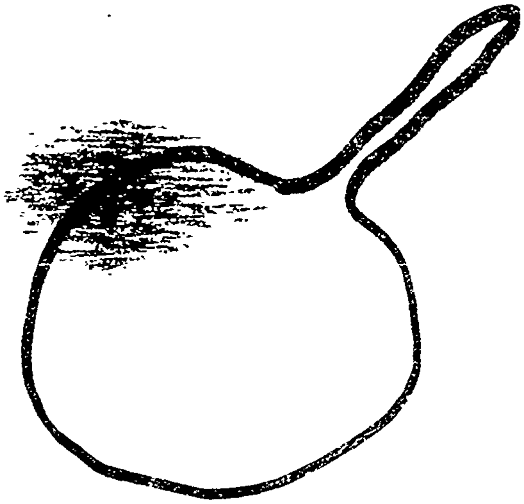
L



FoK

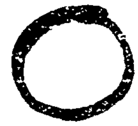


FoL

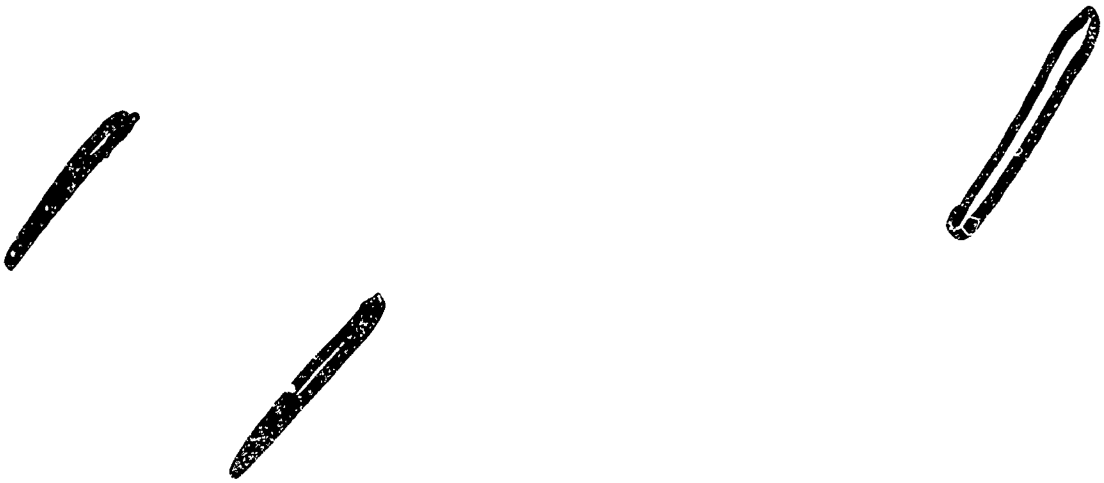


F

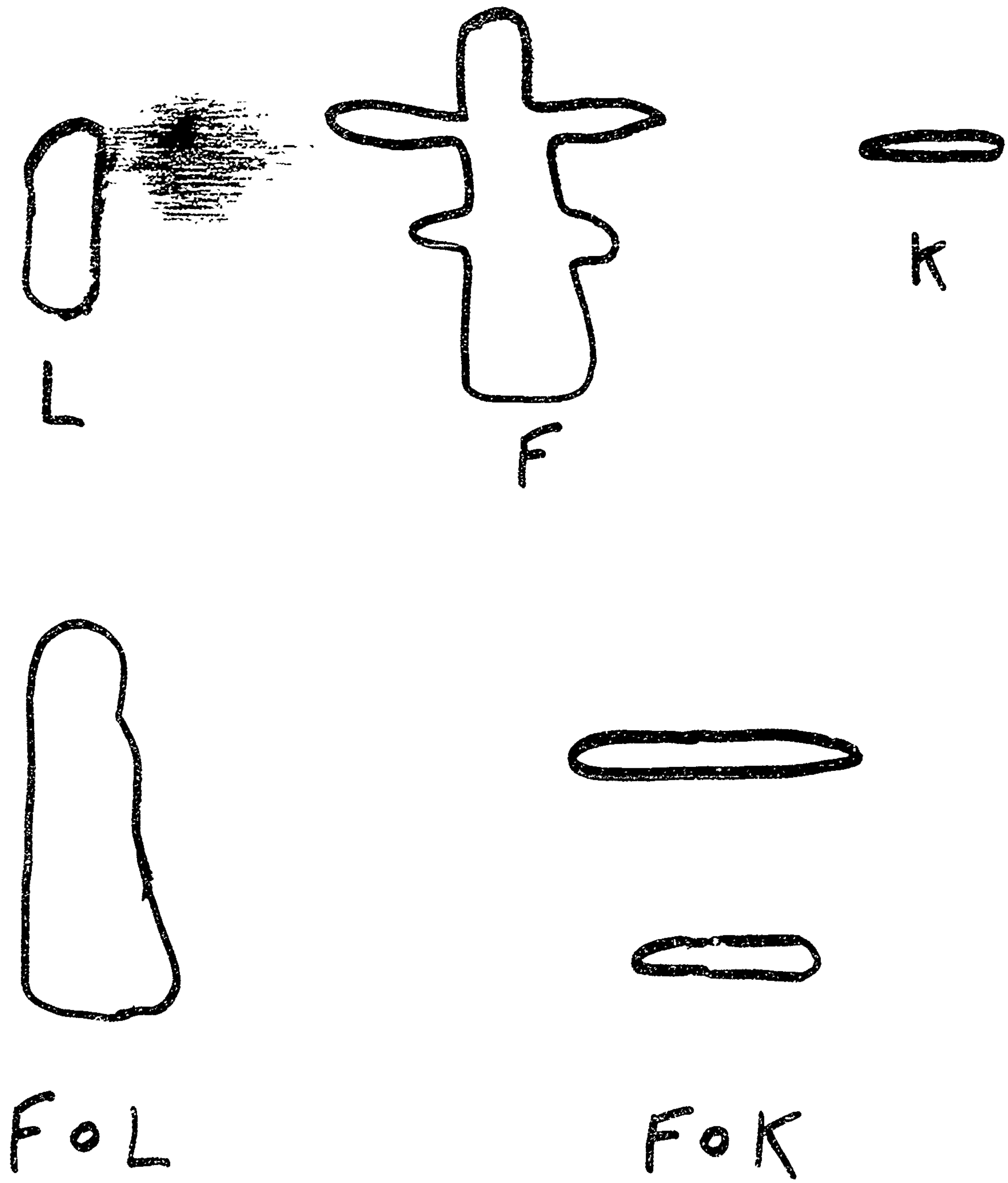
K

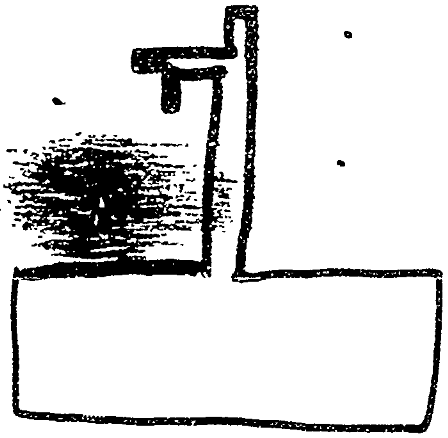


L

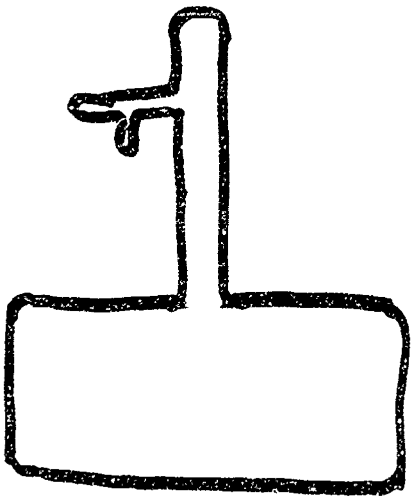


F-FoK F-FoL

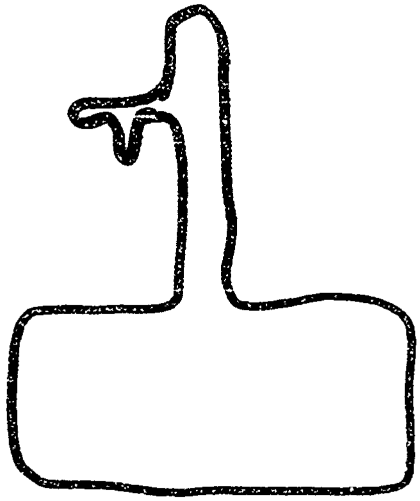

L
F
K
FoL
FoK

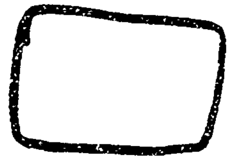


F K_1



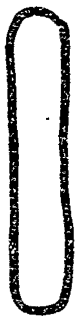
$F \circ K_1$





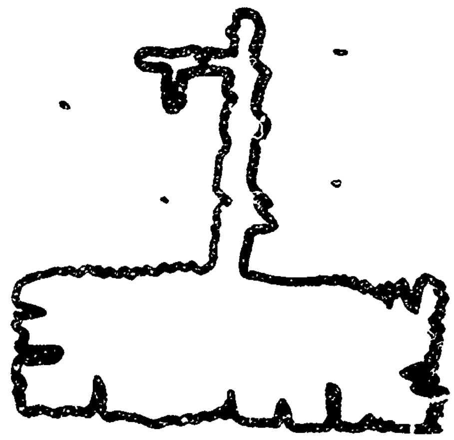
$F \circ K_1$ $\qquad$ K_2

$G = F \circ K_1 - (F \circ K_1) \circ K_2$ $\qquad$ K_3

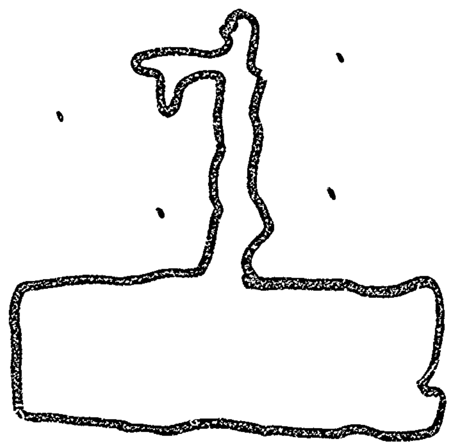


$[F \circ K_1 - (F \circ K_1) \circ K_2] \circ K_3$

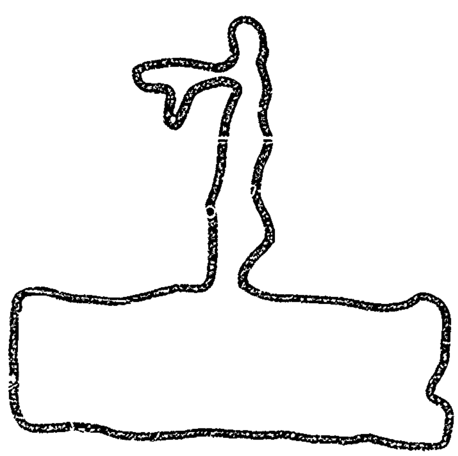
$G \circ K_3$

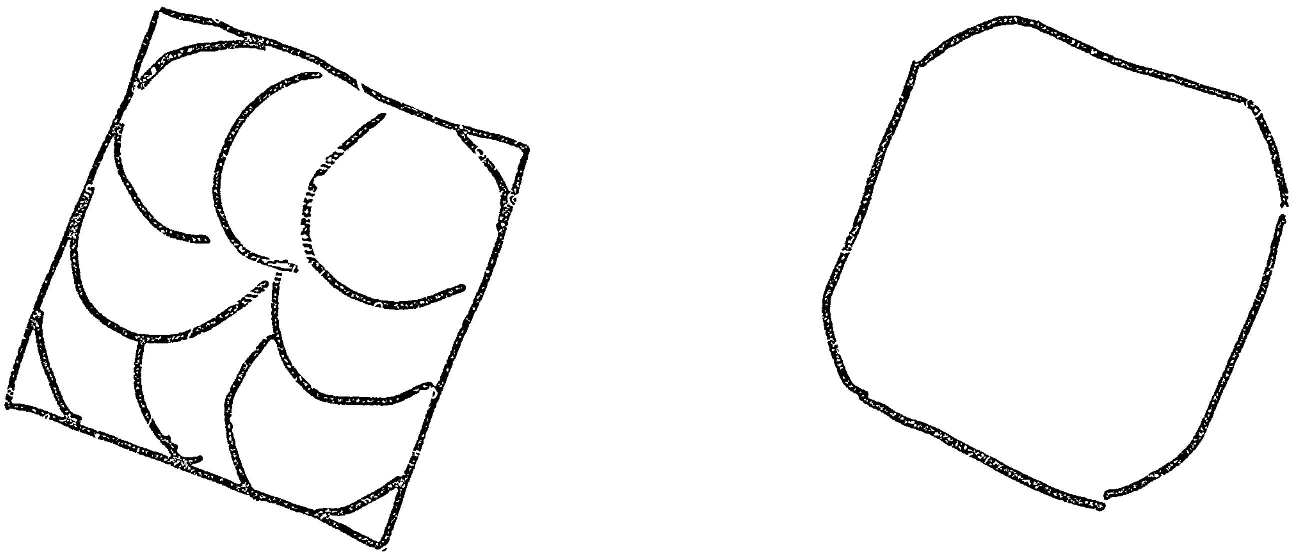


F K$_1$

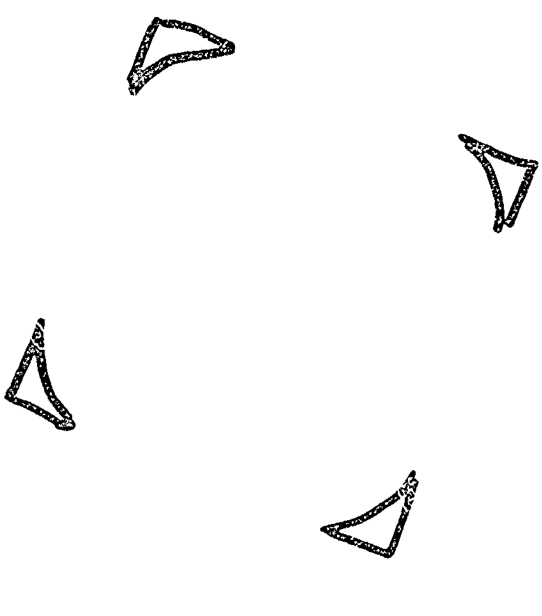


F • K_1

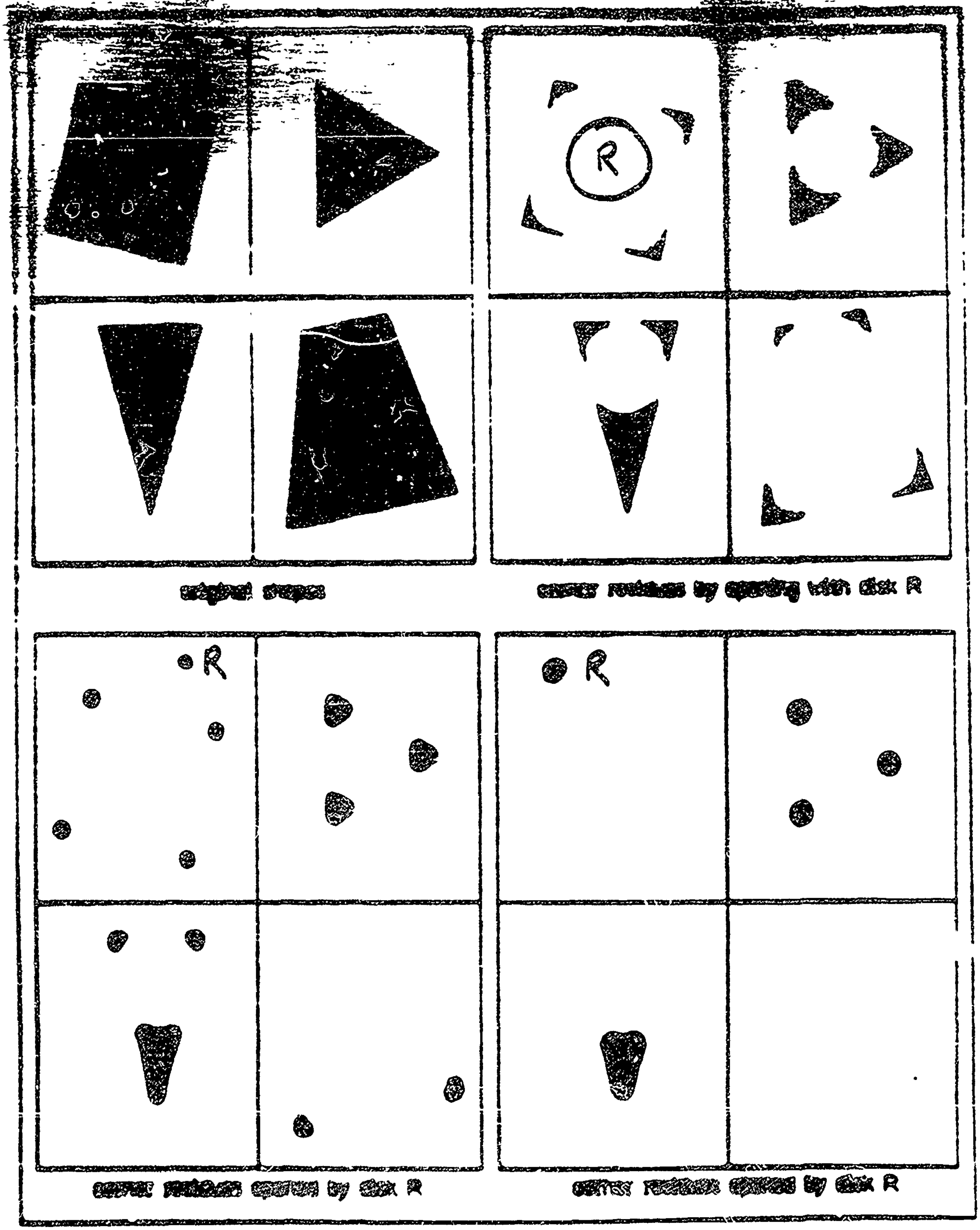




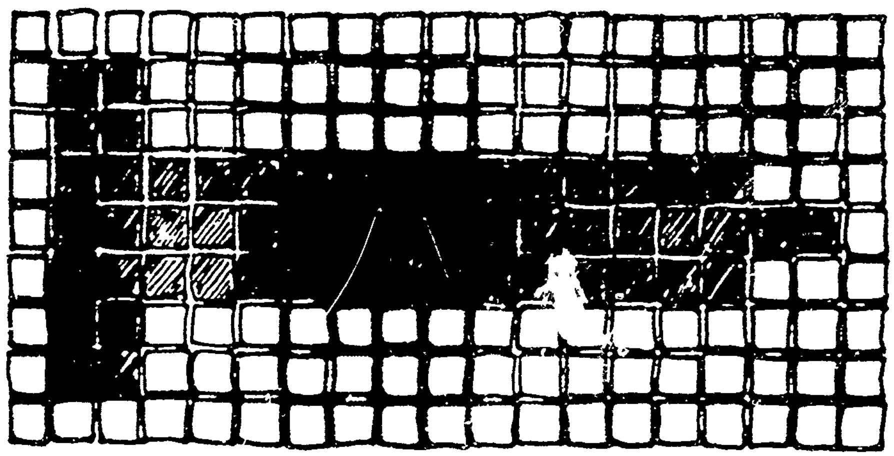
SQUARE
OPENED BY
DISK

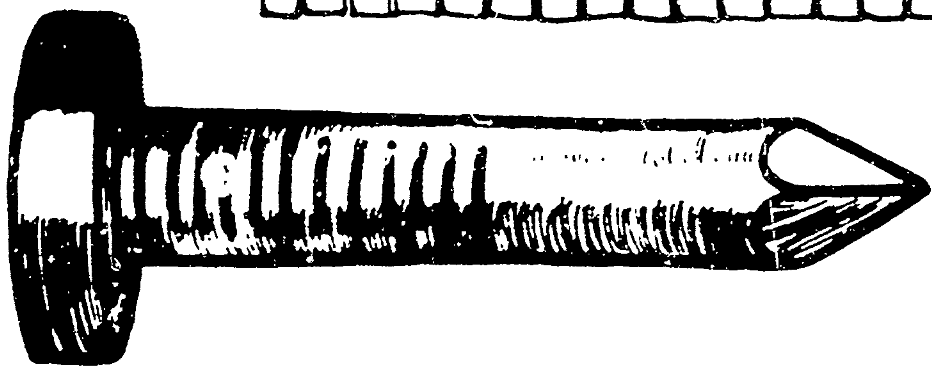


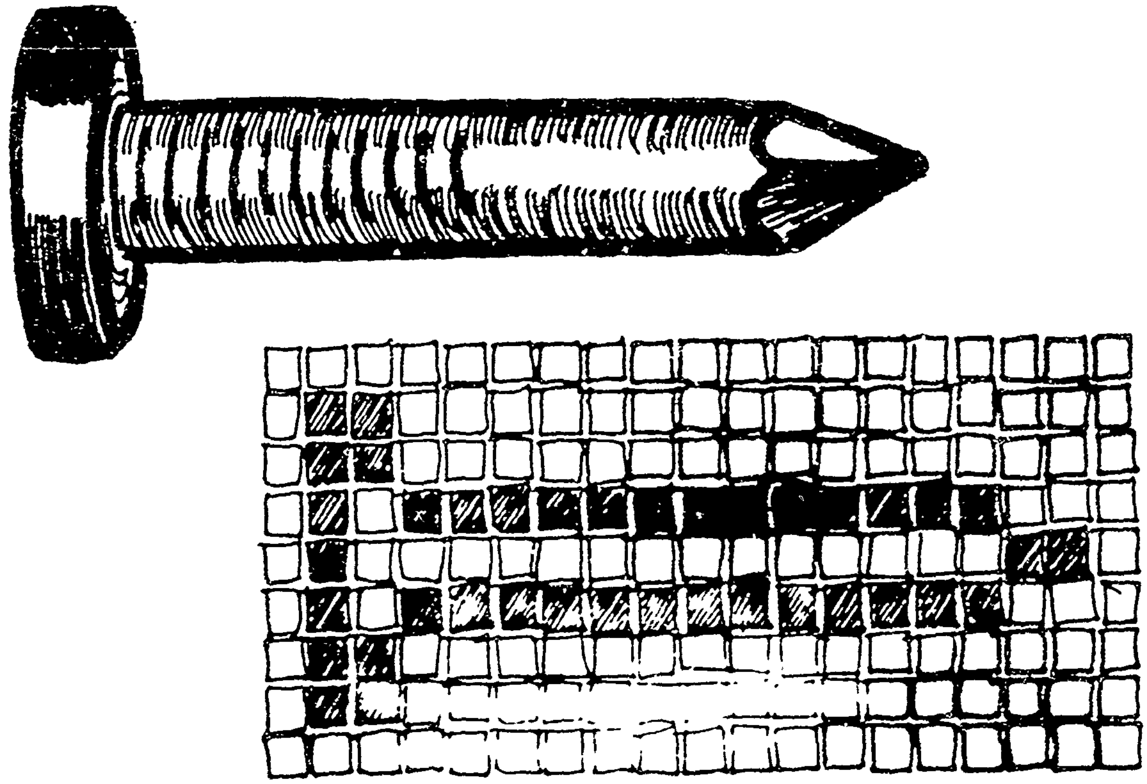
RESIDUE



Discrimination of square and regular triangle by an iterative sequence of morphological openings

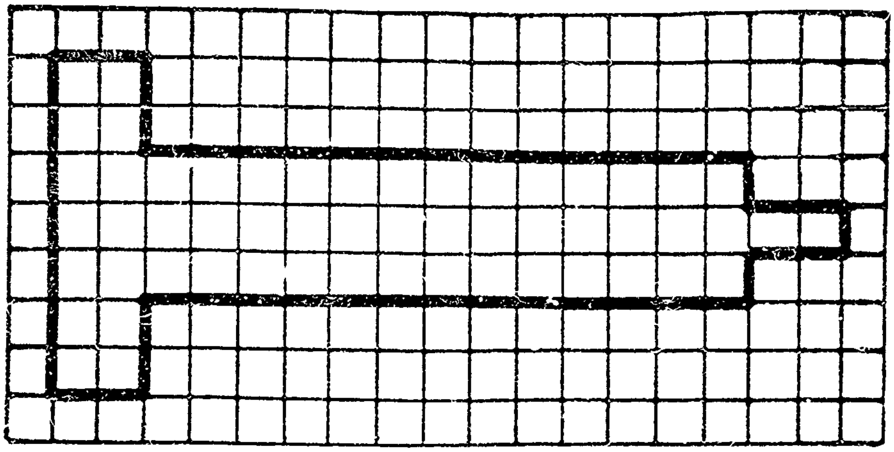


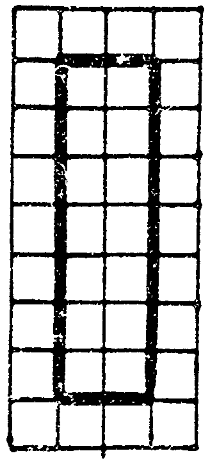




AREA	55
PERIMETER	39
LONGEST HORIZONTAL .	17
LONGEST VERTICAL ...	7
NUMBER OF HOLES	0

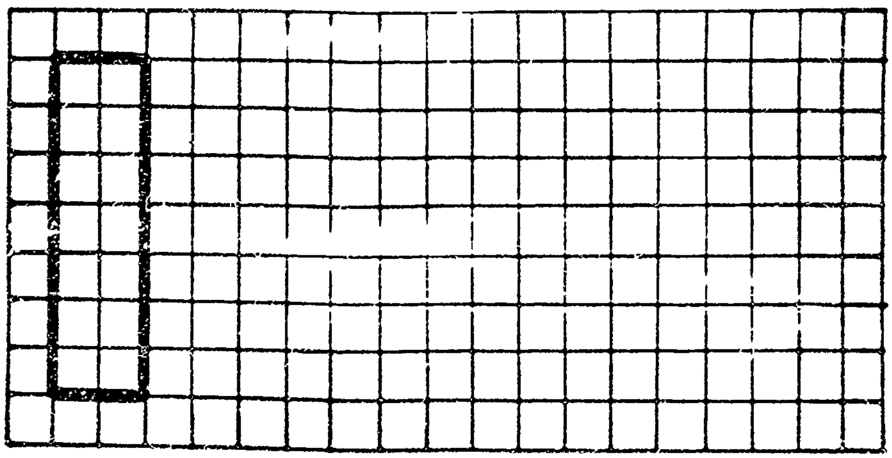
DIGITAL NAIL IMAGE



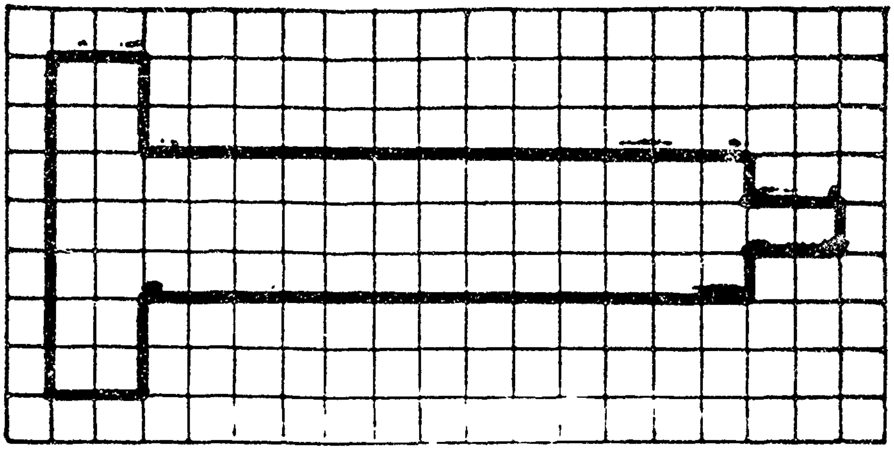


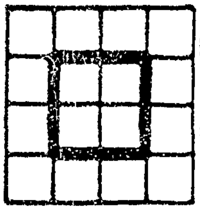
STRUCTURING ELEMENT

MORPHOLOGICAL OPENING



DIGITAL NAIL IMAGE





STRUCTURING ELEMENT

MORPHOLOGICAL OPENING

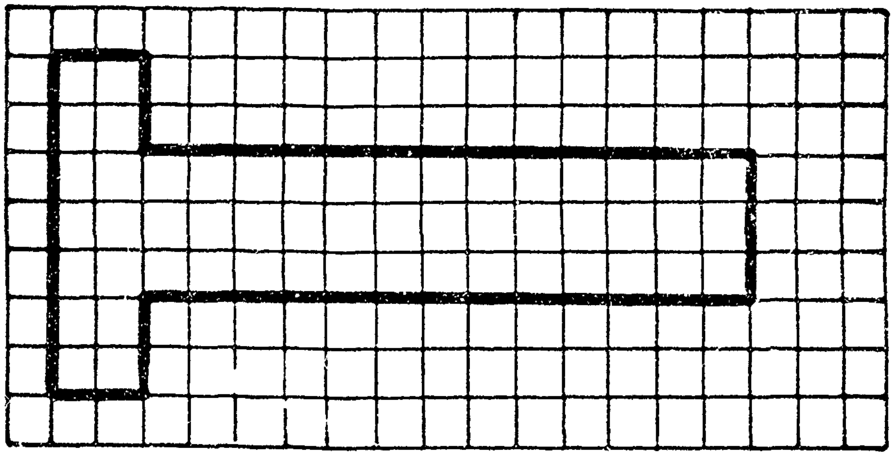
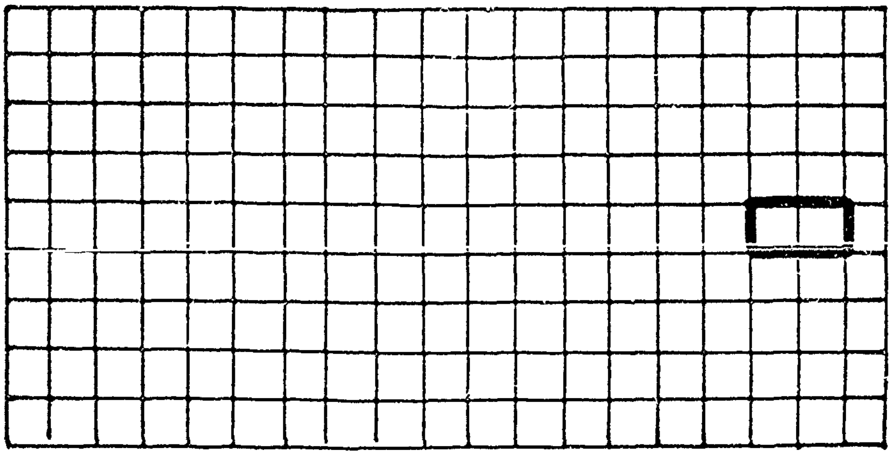
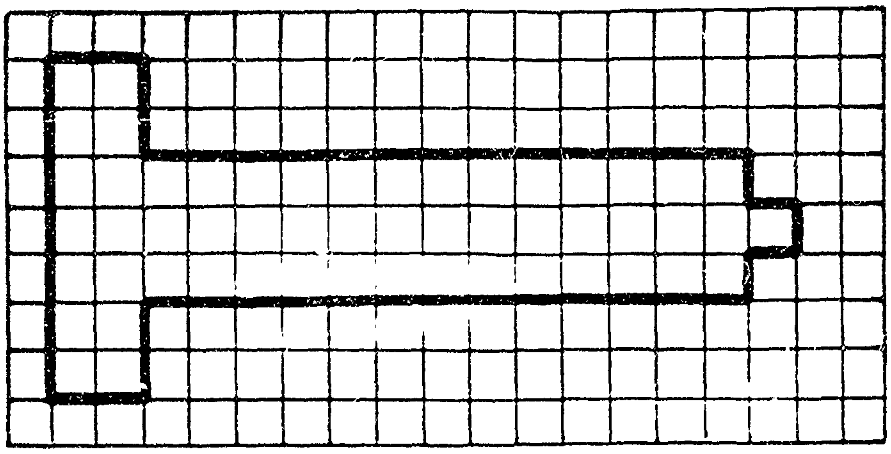


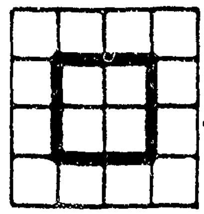
IMAGE TRANSFORMATION



ORIGINAL – OPENING

DEFECTIVE DIGITAL NAIL





STRUCTURING ELEMENT

MORPHOLOGICAL OPENING

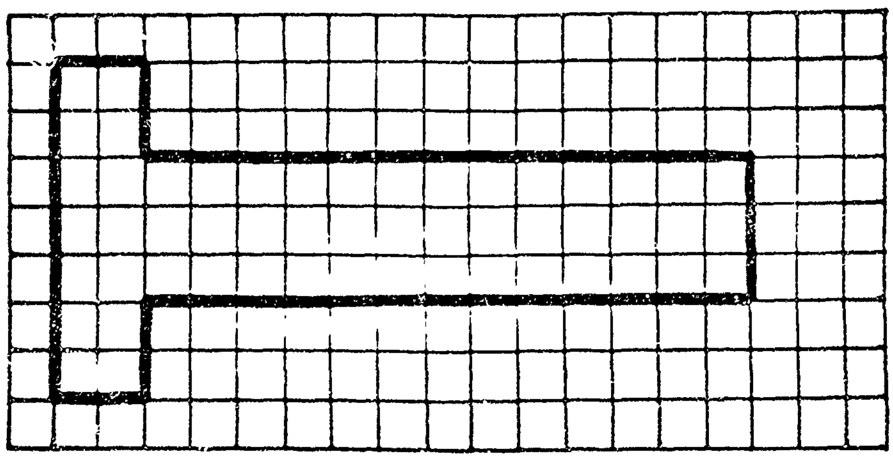
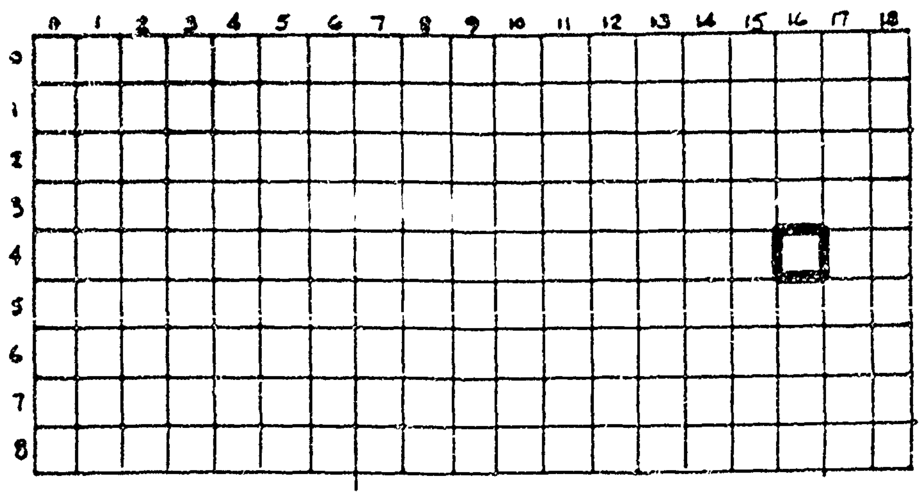
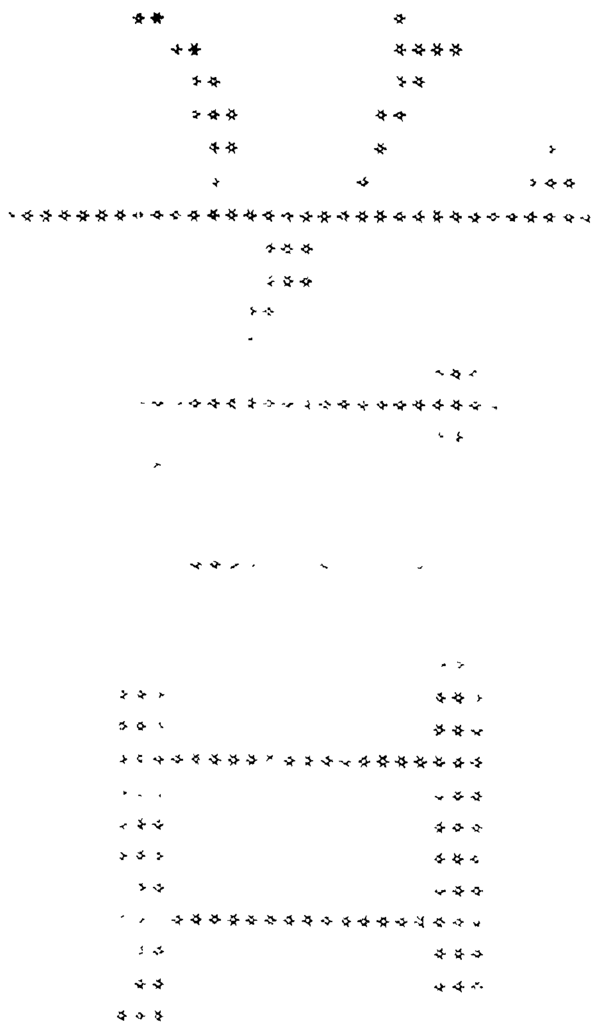


IMAGE TRANSFORMATION



ORIGINAL - OPENING



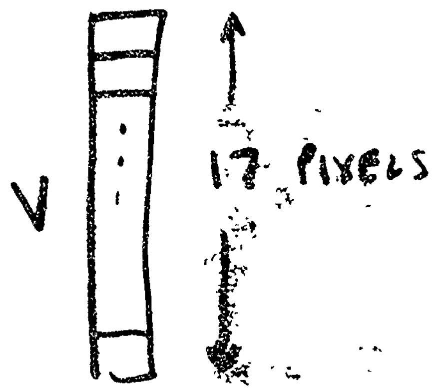
A

SHO AT FIRST

```
***
***
***
***
***
***
```

```
***
***
***
***
***
```

EROSION




```
***
***
***
***
***
***
***
***
***
***
***
***
***
***
***
***
***
***
***
***
***
```

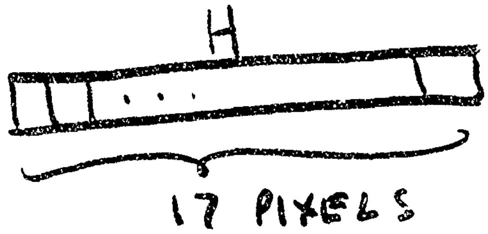
```
***
***
***
***
***
***
***
***
***
***
***
***
***
***
***
***
***
***
***
```

OPENING

A∘V

```
****************
```

EROSION



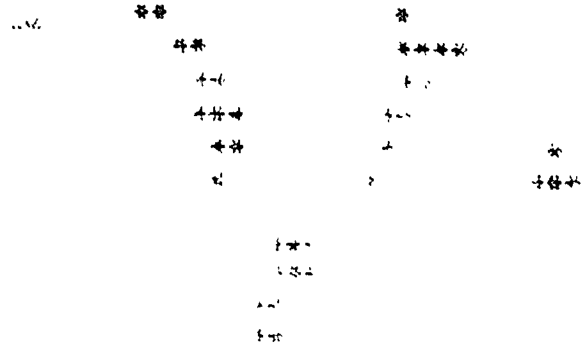

```
*****

****

****

****
```

```
********************************
```

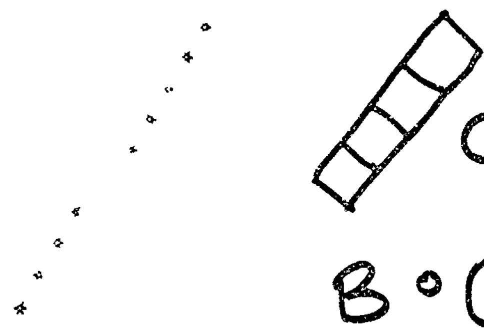
OPENING

$A \circ H$

```
*********************

********************

********************

********************
```

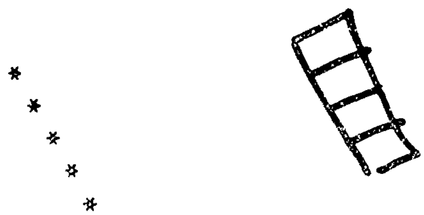


$$B = A \cap [(A \circ H) \cup (A \circ V)]^c$$



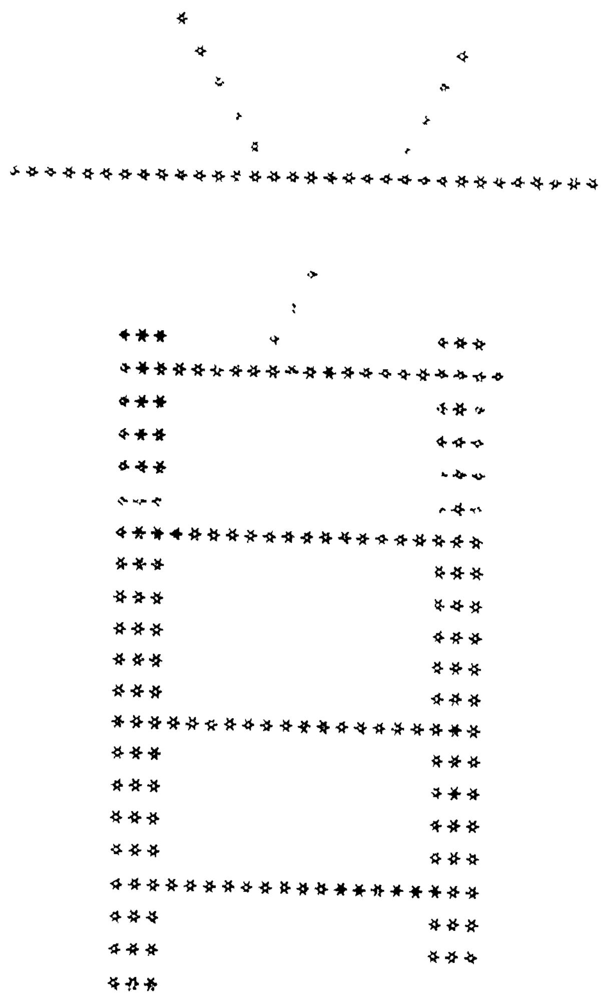
C

$B \circ C$



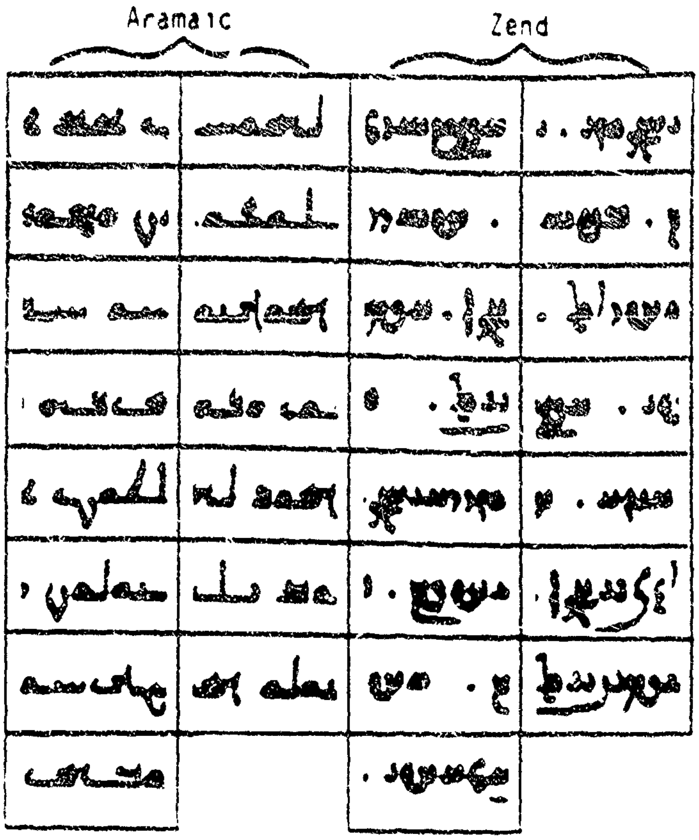
D

$B \circ D$

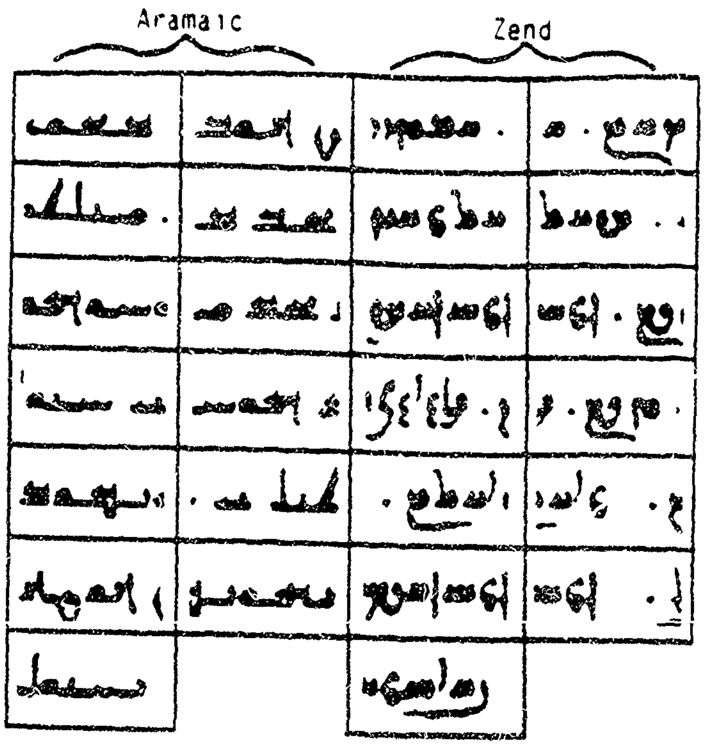


$$(A \circ H) \cup (A \circ V) \cup (B \circ C) \cup (B \circ D)$$

$$B = A \cap [(A \circ H) \cup (A \circ V)]^c$$



TRAINING



TEST

ARAMAIC SCRIPT HAS LONG HORIZONTAL LINES

DEFINE HL TO BE A LONG HORIZONTAL LINE STRUCTURING ELEMENT

$$I \ominus HL$$

BUT SOME ZEND SAMPLES CONTAIN LONG HORIZONTAL LINES

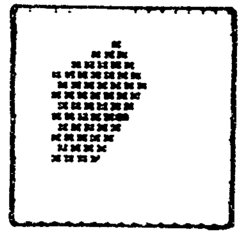
NOW UPPER CONTOUR OF ZEND SCRIPT IS JAGGED HORIZONTAL LINES OF ARAMAIC OFTEN OCCUR BELOW EMPTY SPACES

$$\underbrace{(I-HL)}_{\text{IDENTIFY HORIZONTAL LINES}} \cap \Big[\underbrace{(I^c \ominus B)}_{\text{IDENTIFY BACKGROUND EMPTY SPACE}} \underbrace{\oplus\, C}_{\text{EXTEND TO BELOW}}\Big]$$

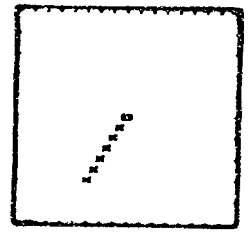
Feature Detector Program Form: $(I \ominus S1) \cap ((\sim I \ominus S2) \oplus S3)$

Structuring Elements:

$S1 = 3*E122$



$S2 = 4*E150 \oplus 3*E121$



$S3 = 6*E110$

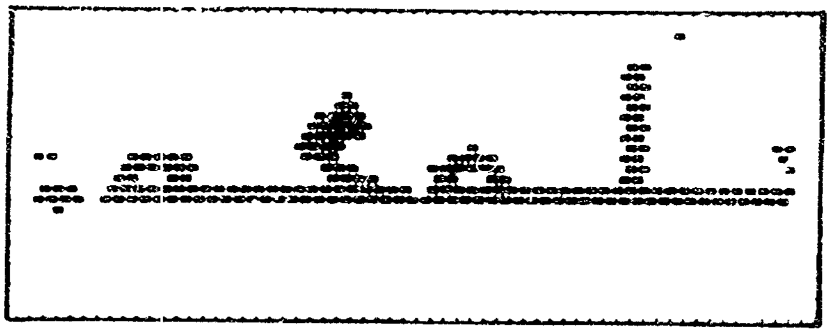


Image Sample from Aramaic Script

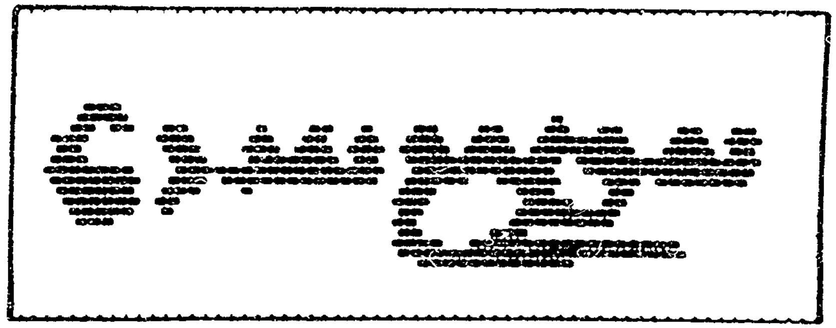
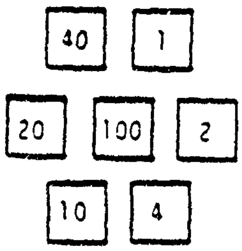
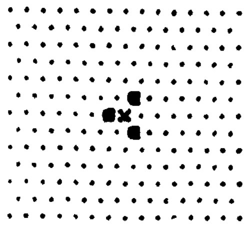


Image Sample from Zend Script

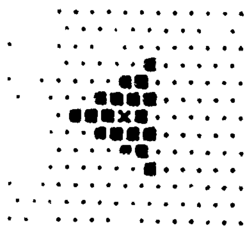


Numbering of points in the elemental window.



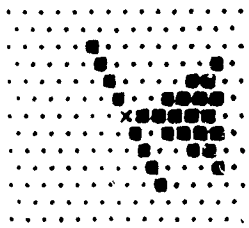
E125

Elemental structuring elements are numbered by adding the numbers for each point which is included.



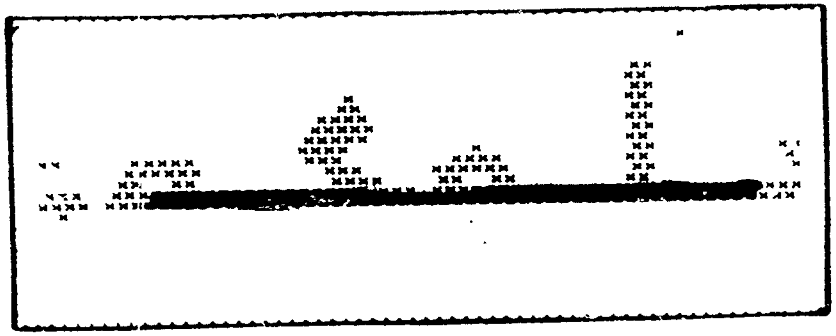
3*E125

Repetition factors denote dilating an elemental structuring element by a number of times.

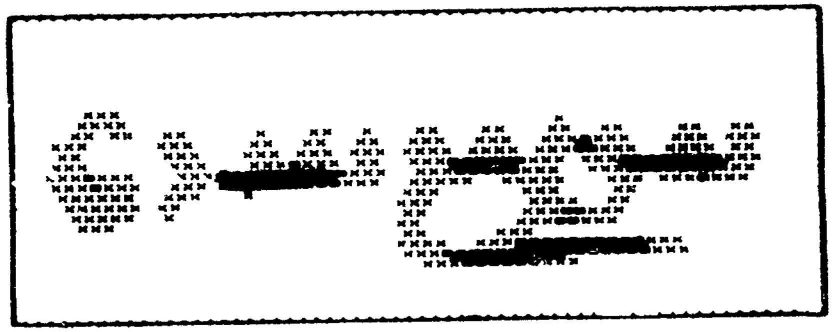


$(4*E144) \cup (3*E125 \odot 4*E002)$

Arbitrary structuring elements are built from elemental structuring elements by dilation and union.

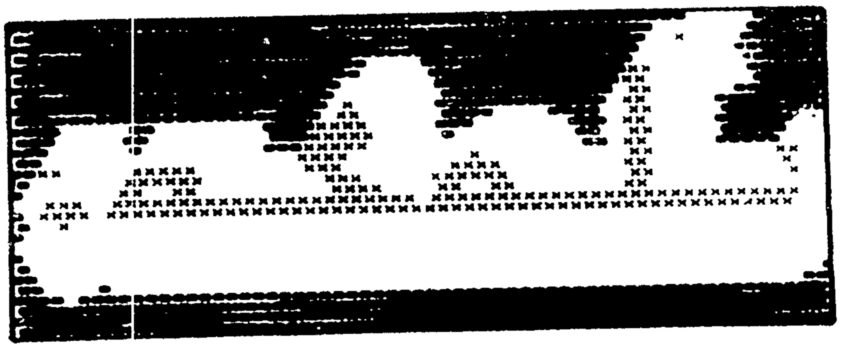


Result of Step 1 on Aramaic Sample

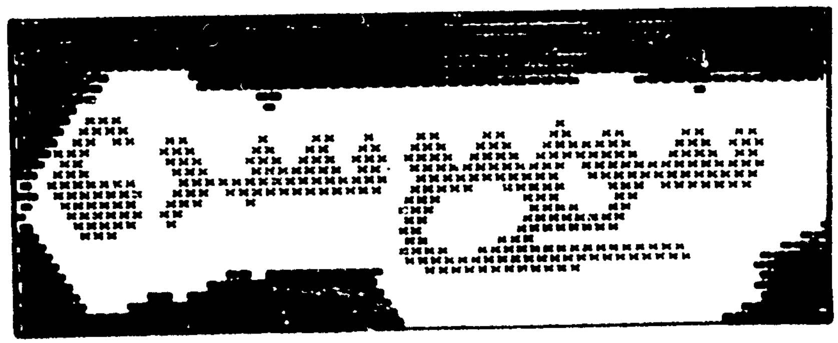


Result of Step 1 on Zend Sample

Step 1: $(I \ominus S1)$

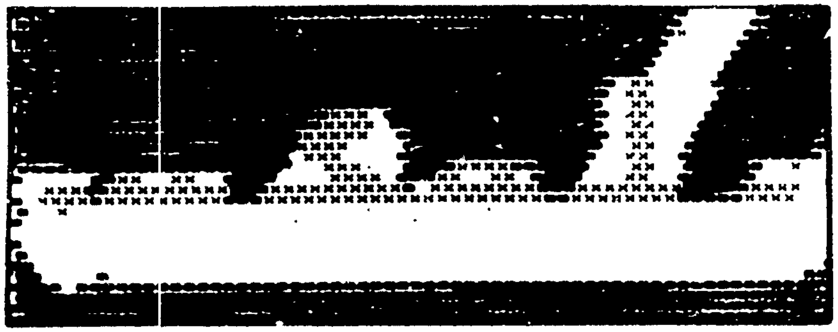


Result of Step 2 on Aramaic Sample

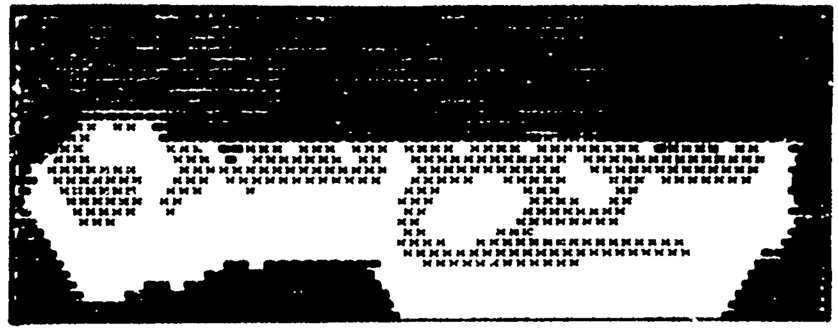


Result of Step 2 on Zend Sample

Step 2: $(\sim I \ominus S2)$

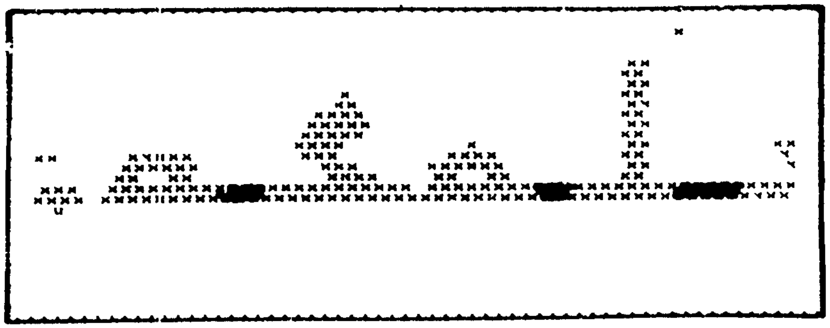


Result of Step 3 on Aramaic Sample

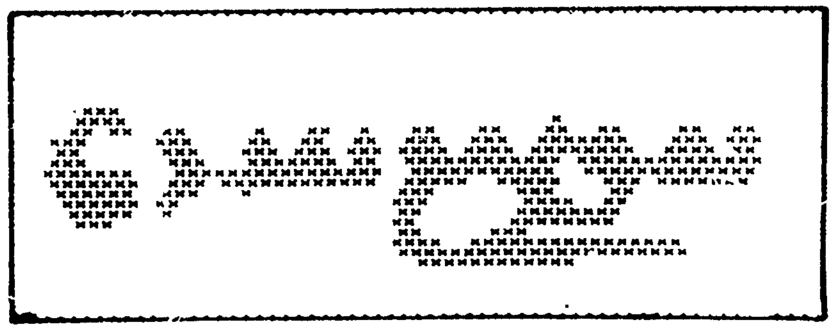


Result of Step 3 on Zend Sample

Step 3: $((\sim I \ominus S2) \oplus S3)$



Result of Step 4 on Aramaic Sample



Result of Step 4 on Zend Sample

Step 4: $(I \ominus S1) \cap ((\sim I \ominus S2) \oplus S3)$

$$(X \ominus S1) \cap \Big(\underbrace{(C \oplus S2)}_{\text{SHIFT}} \underbrace{\oplus\, S3}_{\text{EXTEND}}\Big)$$

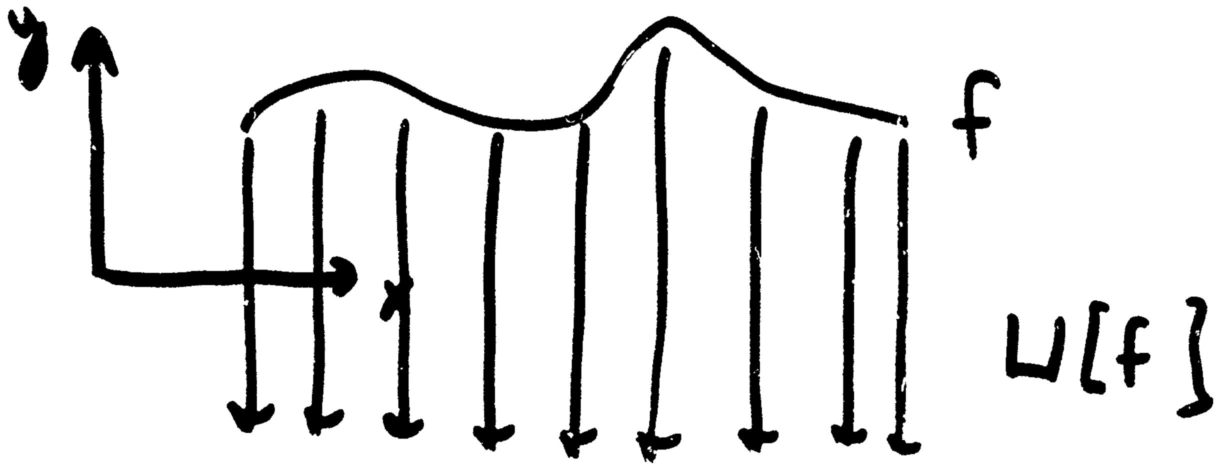
$$X = \begin{cases} I \\ I \bullet \text{Disk}(\text{radius} = 8) \ominus \text{Disk}(\text{radius} = 8) \end{cases}$$

$$C = \begin{cases} I \\ I \bullet \text{Disk}(\text{radius} = 2) \end{cases}$$

UMBRA

DEF LET $F \subseteq E^{N-1}$ AND $f: F \to E$. THE UMBRA OF f, DENOTED $\sqcup[f]$, $\sqcup[f] \subseteq F \times E$, IS DEFINED BY

$$\sqcup[f] = \{(x,y) \in F \times E \mid y \leq f(x)\}$$



TOP SURFACE

DEF: LET $A \subseteq E^N$ AND

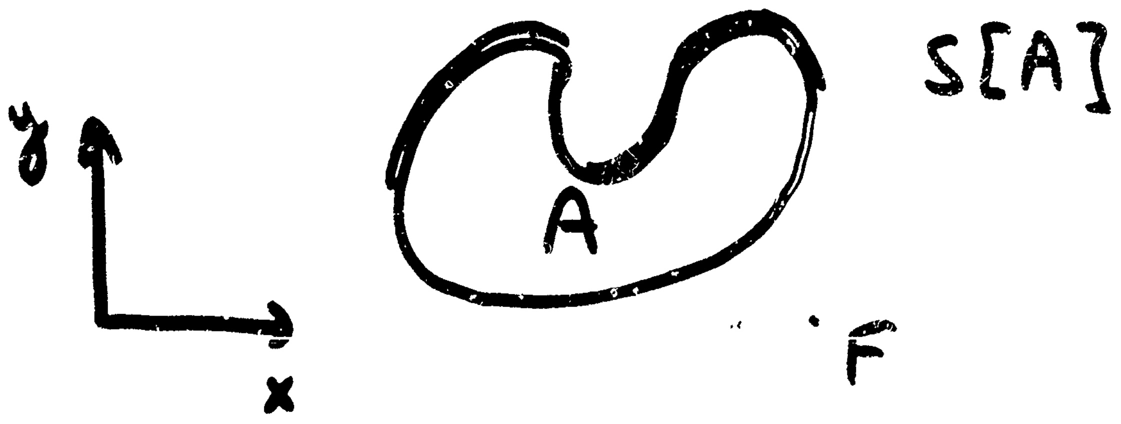
$F = \{ x \in E^{N-1} \mid \text{FOR SOME } y \in E \ (x, y) \in A \}$

THE SURFACE OF A IS DENOTED BY $S[A]$,

$$S[A] : F \to E,$$

AND IS DEFINED BY

$$S[A](x) = \text{MAX}\{ y \mid (x, y) \in A \}$$



GRAY SCALE DILATION

DEF: LET $F, K \subseteq E^{N-1}$ AND $f: F \to E$ AND $k: K \to E$. THE GRAY SCALE DILATION OF f BY k IS DENOTED BY $f \oplus k$, $f \oplus k: F \oplus K \to E$, AND IS DEFINED BY

$$f \oplus k = S[\, U[f] \oplus U[k] \,]$$

GRAY SCALE DILATION

$$f \oplus g = g \oplus f$$

$$(f \oplus g) \oplus h = f \oplus (g \oplus h)$$

$$\mathrm{MAX}\{f, g\} \oplus h = \mathrm{MAX}\{f \oplus h, g \oplus h\}$$

$$(\alpha f) \oplus (\alpha h) = \alpha (f \oplus h), \quad \alpha \geq 0$$

GRAY SCALE EROSION

DEF: LET $F \subseteq E^{N-1}$ AND $K \subseteq E^{N-1}$.

LET $f: F \to E$ AND $k: K \to E$.

THE GRAY SCALE EROSION OF f BY k IS DENOTED BY $f \ominus k$, $f \ominus k: F \ominus K \to E$, AND IS DEFINED BY

$$f \ominus k = S[\,U[f] \ominus U[k]\,]$$

GRAY SCALE EROSION

$$f \ominus (g \oplus h) = (f \ominus g) \ominus h$$

$$\mathrm{MIN}\{f, g\} \ominus k \doteq \mathrm{MIN}\{f \ominus k, g \ominus k\}$$

$$f \ominus \mathrm{MAX}\{g, h\} \leq \mathrm{MIN}\{f \ominus g, f \ominus h\}$$

$$(\alpha f) \ominus (\alpha h) = \alpha (f \ominus h) \quad \alpha \geq 0$$

$$(f \circ k) \circ k = f \circ k$$

$$(f \bullet k) \bullet k = f \bullet k$$

$$-(f \circ k) = (-f) \bullet \check{k}$$

$$-(f \oplus k) = (-f) \ominus \check{k}$$

PROPERTIES

$$S[U[f]] = f$$

$$U[f \oplus g] = U[f] \oplus U[g]$$

$$U[f \ominus g] = U[f] \ominus U[g]$$

$$U[\mathrm{MAX}\{f, g\}] = U[f] \cup U[g]$$

$$U[\mathrm{MIN}\{f, g\}] = U[f] \cap U[g]$$

$$S[A \cup B](x) = \mathrm{MAX}\{S[A](x), S[B](x)\}$$

$$S[A \cap B](x) \leq \mathrm{MIN}\{S[A](x), S[B](x)\}$$

CONVOLUTION

$$f * g = g * f$$

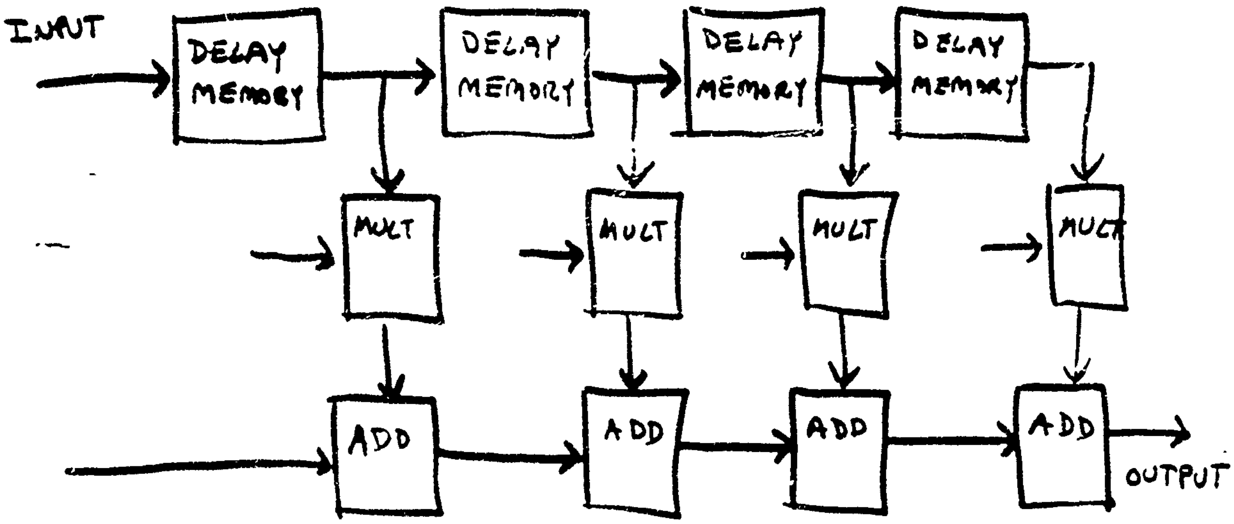
$$(f * g) * h = f * (g * h)$$

$$(f + g) * h = f * h + g * h$$

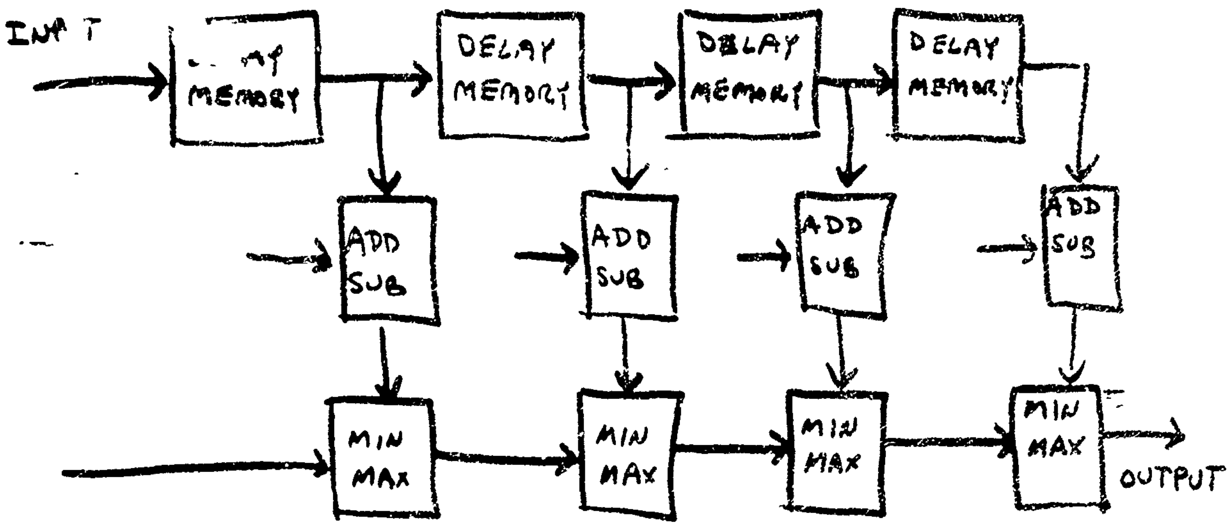
$$(\alpha f) * h = \alpha (f * h)$$

$$(f \oplus k)(x) = \max_{\substack{z \in K \\ x-z \in F}} \{ f(x-z) + k(z) \}$$

$$(f \ominus k)(x) = \min_{z \in K} \{ f(x+z) - k(z) \}$$



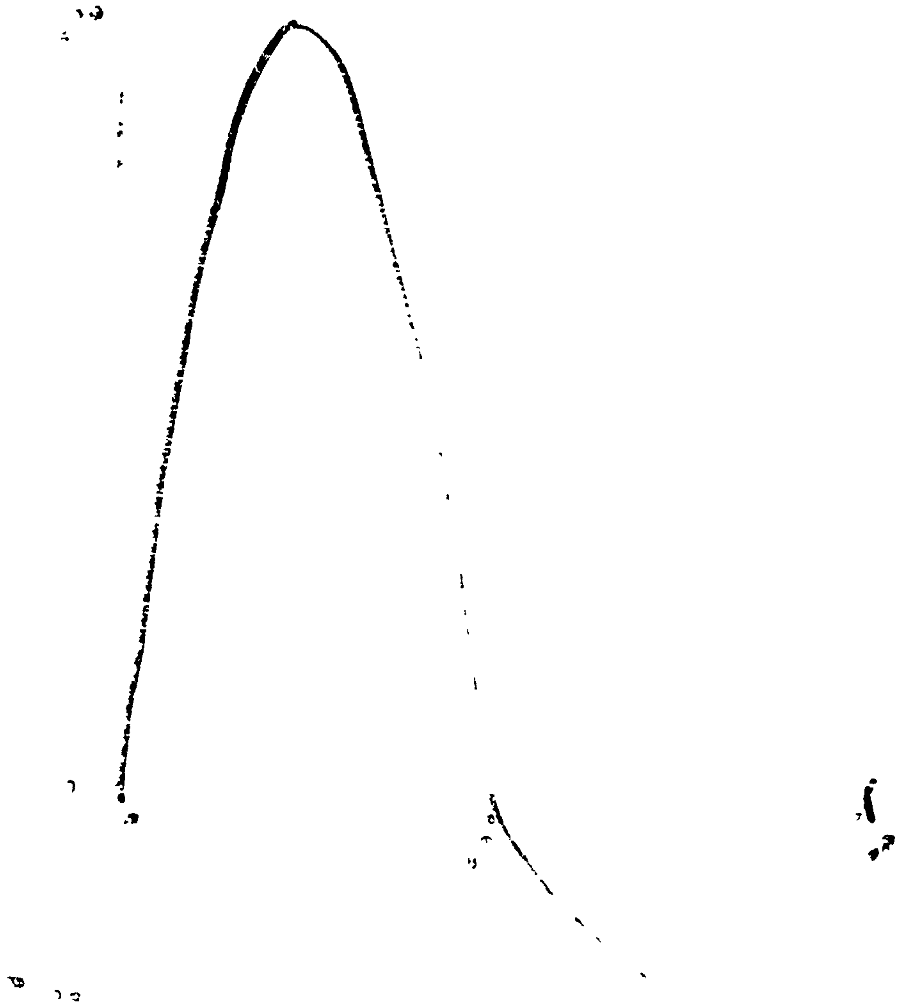
INPUT
DELAY MEMORY
DELAY MEMORY
DELAY MEMORY
DELAY MEMORY
MULT
MULT
MULT
MULT
ADD
ADD
ADD
ADD
OUTPUT



INPUT
DELAY MEMORY
DELAY MEMORY
DELAY MEMORY
DELAY MEMORY
ADD SUB
ADD SUB
ADD SUB
ADD SUB
MIN MAX
MIN MAX
MIN MAX
MIN MAX
OUTPUT

$$100 \sin \frac{x\pi}{10}$$

$$P(x) = 25 - x^2 \quad -5 \le x \le 5$$



$$P(x) = 25 - x^2 \quad -5 \le x \le 5$$

$$100 \sin \frac{\pi x}{10} \cdot P(x)$$